Welcome

Welcome to the SAT Math portion of the Prep Expert Course! When I was in high school, SAT Math was always the section that I scored the highest in. However, even if SAT Math is your worst section, do not worry. We will teach you many effective strategies to ace this section.

SAT Math is very important to do well in. Not only does it make up half of your total SAT score, but many colleges will judge your quantitative skills based on your SAT Math score.

Black & White

Many students believe the SAT Math section is objective and that the SAT Reading and SAT Writing sections are subjective. However, all sections of the SAT are objective. There are no questions that are open to interpretation. Nevertheless, the SAT Math section may not be as straightforward as you think. SAT Math question writers often use phrase questions in a certain way in order to trick students. We will show you all of the ways that they will try to trick you.

High School Math

In order to fully increase your SAT Math score, you must change the way you currently think during this section of the exam (unless you are already getting a perfect score). Because of high school math classes, we have been indoctrinated to think about math problems in certain ways.

In high school, we are often given exams that are not multiple-choice. Therefore, we never think about how we can use the answer choices to our advantage when we are given a mutiple-choice test.

We will show you how to use the multiple-choice nature of the SAT to your advantage. This will require you to rethink how you do many math problems. Often, it will not be the same way that you are used to doing problems in your high school math classes.

2400 SAT Math

Two of the biggest differences between the New SAT Math section and the old SAT Math section are the addition of a no-calculator section and more advanced topics in math.

First, many students are afraid that the SAT Math section now includes a no-calculator portion.

However, Prep Expert students should have no fear. I actually believe the calculator is a handicap for many students. I will explain why later, but I have always advocated that students keep calculator use to a minimum on the SAT. So it's actually great news that you can't use a calculator for certain portions of the SAT Math section! This will force you to practice doing problems without a calculator. The more you practice doing problems without a calculator the more comfortable you will be answering questions on the calculator-allowed portion of the SAT Math section without a calculator. This will actually increase your SAT Math score!

The other major change to the SAT Math section is the addition of more advanced mathematical topics. For example, trigonometry is now tested on the SAT Math section, but was not tested on the 2400-version of the SAT. However, you only need to know some basic concepts in trigonometry to be able to answer these questions. We will teach you all of the trig concepts you need for the SAT in this course. Therefore, it's not necessary for you to have taken a full trigonometry class prior to studying for the SAT.

SAT MATH

FREQUENTLY ASKED QUES-TIONS

What Is The Format? 1

SAT Reading Format	
Time	80 Minutes
> Calculator Portion (38 Questions)	55 Minutes
> No-Calculator Portion (20 Questions)	25 Minutes
Questions	58 Questions
> Multiple Choice	> 45 Questions
> Student-Produced Response (Grid-In)	> 13 Questions
Content	
> Heart of Algebra	> 19 Questions
> Problem Solving & Data Analysis	> 17 Questions
> Passport to Advanced Math	> 16 Questions
> Additional Topics in Math	> 6 Questions

The third section of every New SAT will be a 25-min no-calculator Math section with 20 questions.

The fourth section of every New SAT will be a 55-min calculator Math section with 38 questions.

Time

In total, there is about the same amount of time dedicated to the SAT Math section on the New SAT as there was on the old 2400-version of the SAT. However, on the old 2400-version of the SAT, the 70 minutes that were dedicated to the SAT Math section were broken up into three sections of 25 minutes, 25 minutes, and 20 minutes.

On the new 1600-version of the SAT, there are two sections that makeup a total of 80 minutes (55 minutes and 25 minutes). This means you must stay focused during that 55-minute section for a much longer period of time. This is not easy, especially since it's easy to make mistakes on SAT Math problems. However, we will give you some techniques on how to stay focused on during such a long section.

Questions

Multiple Choice

These questions offer you 4 possible answer choices. Although there are fewer answer choices than the previous 2400-version of the SAT had, the reduction in answer choices doesn't help you as much on the SAT Math section as it does on the SAT Reading and SAT Writing sections. Students should focus on arriving at the correct solution on the SAT Math section whereas they should focus on eliminating all of the incorrect answers on the SAT Reading and SAT Writing sections.

Whether you have 4 or 5 answer choices on the SAT Math section does not make a big difference. However, when you are guessing on the SAT Math section, the reduction in answer choices can make a difference. But after you learn the Expert Math Strategies in this course, you won't have to do too much guessing.

Student-Produced Response

These questions do not give you multiple answers to pick from. Instead, you must grid-in your solution on the answer sheet. The SAT will accept any way that you write the answer (i.e. as a fraction or as a decimal).

Students often believe these questions are more difficult because they don't have the cushion of answer choices to fall back on. However, your approach to student-produced response questions should be the same as your approach to multiple-choice questions. In addition, student-produced response questions cannot have answers that are negative!

Content

Heart of Algebra

- Equations
- Expressions
- Inequalities
- System of Equations
- Formulas

Problem Solving & Data Analysis

- Ratios
- Proportions
- Percentages
- Graphical Relationships
- Data Summarization

Passport to Advanced Math

- Rewriting Expressions
- Quadratic Equations
- Higher Order Equations
- Polynomials

Additional Topics in Math

- Area & Volume Calculations
- Lines
- Angles
- Triangles
- Circles
- Trigonometric Functions

*** It's worth noting that "Additional Topics in Math" only make up 6 questions on the entire exam. These are also the questions that relate to geometry and trigonometry. Therefore, it really isn't necessary to have gone through an entire course on geometry or trigonometry in order to do well on the SAT Math section, given that so few questions will be related to these topics.**

What's Different? 2

2400 SAT Math	New SAT Math
Multiple Topics	Less Topics
Less Advanced	More Advanced
Calculator in All Sections	No Calculator Section
800 Points	800 Points

One of the major changes the College Board made with respect to the SAT Math section for the New SAT was reducing the number math topics students need to know. As you can see from the previous FAQ, there are a limited number of topics on the New SAT – algebra, data analysis, advanced topics, and additional topics.

On the old version of the SAT, the list would be much longer. This reduction in the number of topics is good news for students. However, the College Board has also increased the depth of knowledge students need to have for each topic. So there is the tradeoff. Fewer topics, but more depth of knowledge required for those topics. We will teach you not only the topics you need to know for the SAT Math section, but also in enough depth that you will be thoroughly prepared for the exam.

Another big change is that there is now a no-calculator portion of the SAT Math section. However, as I already stated, this is probably a good thing. Every math question on the SAT can be solved without a calculator.

In addition, at Prep Expert, we really stress the minimal use of a calculator. In fact, we encourage our students to try to avoid using the calculator as much as possible (I'll explain why later). By having the calculator by their side, students often feel more comfortable doing problems.

However, the calculator cannot help you with the thinking that needs to occur in order to solve problems. Only your mind can do that. And we will show you many test-taking problem solving techniques so that you will no longer feel lost without a calculator.

The score allotment to SAT Math has never changed. On the original 1600-verison of the SAT, Math was worth 800 points. On the 2400-version of the SAT, Math was worth 800 points. On the new 1600-version of the SAT, Math is worth 800 points.

This reflects not only the importance that the College Board places on quantitative skills, but also the importance that many colleges place on math skills. On the 2400-version of the SAT, Math only made up approximately 33% of your overall SAT score. Therefore, you could get away with not doing so well on the SAT Math section and still having a decent overall score.

However, on this new 1600-version of the SAT, Math is worth 50% of your overall SAT score. So it's very important to do well on the section. If you are a student who struggles with math in general, then it is imperative that you learn every strategy we teach for the SAT Math section.

How Is SAT Math Scored? 3

You will receive many different scores on your score report related to SAT Math. However, I would not pay attention to most of them, except your Math Section Score out of 800 and your Total SAT Score out of 1600. But for the sake of completeness, I will review what each of the scores means here.

Subscores (1-15)

Heart of Algebra	This subscore indicates how well you performed on algebra questions.
Problem Solving & Data Analysis	This subscore indicates how well you performed on problem solving and data analysis questions.
Passport to Advanced Math	This subscore indicates how well you performed on the advanced topics in math such as trignometry.

Test Score (10-40)

Math	This test score indicates how well you performed on all of the SAT Math questions.

Cross-Test Score (10-40)

Analysis in Science	This cross-test score indicates how well you performed on science passages and questions across the SAT Writing, SAT Reading, and SAT Math sections.

Analysis in History/Social Science	This cross-test score indicates how well you performed on history/social science passages and questions across the SAT Writing, SAT Reading, and SAT Math

Section Score (200-800)

Math	This section score indicates how well you performed on the SAT Math section overall.

Total Score (400-1600)

Reading & Writing + Mathematics	This is your SAT score out of 1600 that you will remember for the rest of your life.

Is SAT Math Harder 4

Overall, I believe the SAT Math section is about the same level of difficulty as the previous 2400-version of the SAT. Although there is less math content that you need to know for the New SAT, the content that you do need to know needs to be understood in more depth. This tradeoff between amount of content and depth of content essentially cancel each other out.

There are more advanced topics on the SAT Math section than ever before. Previously, the SAT only tested up to Algebra II. But now, the SAT tests trigonometry. However, there is very little trigonometry tested on the New SAT.

If you recall, there are six questions related to advanced topics on the SAT Math section. More likely than not, all six of those questions will not be trigonometry. So you are likely going to encounter only 2-3 trigonometry questions on the New SAT Math section. In addition, the trigonometry tested is not too advanced – sine, cosine, and tangent.

Finally, with the addition of the no-calculator section to the SAT Math section, I believe Prep Expert students have an advantage over other students. Because we train you to solve all problems without a calculator, you will have no problem with this section. Many other students may struggle without a calculator.

Overall, the new SAT Math section is easier if you are a student who typically does well on math because of the reduced content and fewer answer choices. If you are a student who typically struggles with math, the New SAT Math section will likely be harder for you due to the more advanced content that you need to understand in-depth.

No matter what type of student you are, Expert Math Strategies will help you ace this section of the SAT.

How Is SAT Math Different Than High School Math?

SAT Math is different than high school Math. Many students who do well in high school Math often do poorly on SAT Math. This makes perfect sense to me. In high school Math, teachers train students to use a top-down approach. Because you are not given answer choices on most math tests in high school, students never think to use answer choices to their advantage.

But on SAT Math, you are given answer choices. So we need to reengineer the way we think about SAT Math problems using a bottom-up approach. This means using the answer choices to our advantage.

Let me show you an example.

Top-Down Approach (Traditional High School Math)

$$\left(\frac{x}{2}\right) + 3 = 52$$

Without any answer choices in high school, you would solve this problem using the traditional top-down approach:

$$\left(\frac{x}{2}\right) = 49$$

$$x = 98$$

$$\left(\frac{x}{2}\right) + 3 = 52$$

Bottom-Up Approach (Using Answer Choices to Our Advantage)

(A) 49

(B) 55

(C) 98

(D) 110

Plug in 98:

$$\left(\frac{98}{2}\right) + 3 = 52$$

While the top-down approach may seem simpler on this easy problem, we will find that the bottom-up approach may be useful on harder problems. The example above is not to illustrate the advantages of the bottom-up approach just yet (as I agree that the traditional top-down approach is probably easier

in this scenario), but to show you that we need to reengineer our traditional way of thinking about mathematics in order to take advantage of the multiple-choice nature of the SAT.

SAT MATH
EXPERT PRINCIPLES

CAPITALIZE ON COMPLEXITY	1
PUT PENCIL TO PAPER	2
LABEL EVERYTHING	3
HIGHLIGHT QUESTION	4
VERIFY EACH STEP	5

Capitalize On Complexity? 1

Just as we do for every other section of the SAT, we will cover 20 Expert Strategies to help you ace SAT Math. However, I have also created 5 Expert Principles for this portion of the exam. These principles are applicable to the entire SAT Math section and not just to specific problem types. You should keep these principles in mind as you work through all SAT Math problems.

The first Math Expert Principle is Capitalize on Complexity. SAT Math problems are ordered from easy to hard. At the beginning of a section, you will encounter easier problems. At the end of a section, you will encounter harder problems. Make sure you have enough thinking power left at the end of an SAT Math section to tackle the most difficult questions.

You should keep in mind that the order of difficulty resets when you get to the student-produced response questions at the end of each SAT Math section. For example, on the no-calculator 20-question SAT Math section, the order of difficulty will go from easy to hard on the first 15 multiple-choice questions. Then, the order of difficulty will go from easy to hard on questions 16 through 20 – the student-produced response questions.

We should capitalize on the ordered complexity of the SAT Math section to help us score higher on this part of the exam. But first we need to understand what makes an SAT question "easy" or "hard." SAT Math questions are considered "easy" or "hard" based on the number of students that answer a particular question correctly. SAT Math difficulty will follow these guidelines:

- Easy Questions – Most students answer these questions correctly.
- Medium Questions – About half of students answer these questions correctly.
- Hard Questions – Most students answer these questions incorrectly.

Knowing the above information can help us in capitalizing on the complexity of the SAT Math questions. For example, if an easy question is taking you an extraordinarily long time to solve or you are starting to use very complex calculations, you should stop. Since you know that the problem is an easy question that most students answer correctly, you are likely doing something wrong. Try going back to the beginning of the problem and solving it in a way that you might not have originally seen.

In addition, when guessing on an easy SAT Math question, you should be aware that the answer will be somewhat "obvious." Obvious answers are ones that can be easily derived from the numbers or variables in the question.

On the other hand, if a hard question only took you a couple of seconds to solve, then you should stop. Since you know that most students answer hard questions incorrectly, then you may have done something wrong. Hard questions typically require students to put in more time and effort in order to solve them. Try going back to the beginning of the problem and seeing if there was something that you originally missed.

Of course, sometimes using Math Expert Strategies will help you solve hard SAT Math questions extremely fast. Because Math Expert Strategies are not the traditional high school ways to solve SAT Math problems, the SAT is rewarding you for your unconventional problem solving skills.

In addition, when guessing on a hard SAT Math question, you should be aware that the answer will likely be non-obvious. non-obvious answers are ones that cannot be easily derived from the answer choices. Often times, these answer choices are "ugly" looking. They look like no one could ever derive that answer choice; however, it may be to your advantage to pick these complex-looking answer choices when you are guessing on hard SAT Math problems.

Put Pencil To Paper 2

You may recall that Put Pencil to Paper is actually General Expert Strategy #10. However, I am reemphasizing it here as a Math Expert Principle because I believe it is particularly essential to scoring high on the SAT Math section. Putting Pencil to Paper means writing down every detail as you work through the SAT. This concept is more important on SAT Math than any other part of the SAT. Your biggest weapon on the SAT Math section is not your calculator. It is your pencil.

Your mind has two aspects: working memory and thinking power. Working memory and thinking power are inversely related. The more that you use your working memory, the less thinking power you will have.

By Putting Pencil to Paper, your mind no longer has to store so much information in its working memory. You have more thinking power to accurately solve problems. For example, do not try to remember that $x = 2$ or that answer choices B and D are definitely incorrect. Instead, simply write down that $x = 2$ and cross out that answer choices B and D. Details such as these do not need to take up space in your working memory.

On SAT Math, it is imperative to Put Pencil to Paper on every problem. There are so many small details on each SAT Math question that you don't even realize you are storing in your mind. For example, if a problem states that the slope of a certain line is 2, then you should put your pencil to paper and actually label that line as having a slope of 2. You could just leave the line unlabeled and remember that the slope is 2. However, this approach will take up working memory and therefore
decrease your thinking power.

You will be pleasantly surprised at how powerful this technique is on the SAT Math section. Problems that you initially thought were unsolvable suddenly become immediately obvious. Write down as much as you can on the SAT Math section. Draw diagrams, label diagrams, write out equations, substitute numbers, circle unknowns, etc. The more that you keep your pencil glued to the test booklet, the higher your SAT Math score will go. Think with your pencil!

Label Everything 3

Similar to Putting Pencil to Paper, this Math Expert Principle Label Everything also frees up our thinking power during the SAT Math section. When there is a diagram, chart, or other picture on the SAT Math section, make sure you label everything you can.

Although this principle is very similar to the previous one, I created a separate Expert Principle here because it's so important to label items on SAT Math. Yet when I see many students' SAT exams after they have taken the test, I see that they have not labeled all of the given diagrams on the SAT Math section.

Not only should you label what is given to you by the question, but you should over-label. "Overlabeling" means labeling everything possible about a diagram – even if the SAT does not specifically give you that information in a problem.

If you are given the side lengths of the triangle, you should label the lengths of each of the sides. This is a no-brainer. However, to take your labeling one step further, you should also label the height of the triangle. Even though the SAT may not specifically ask about the height of the triangle, you should still label it. The height of a triangle is often used to find the area of the triangle, which is a common item that is needed to solve geometry questions related to triangles. Having the foresight to label items that are not already given to you on SAT Math diagrams is what separates good SAT Math scores from great SAT Math scores.

You should also remember to label neatly. Although this may sound simple, many students miss questions because of sloppy labeling. In addition, make sure to note diagrams that are not drawn to scale. If the diagram doesn't specifically say, "Figure Not Drawn to Scale," then you can assume that the diagram is drawn to scale. Label according to whether it is or isn't drawn to scale.

Highlight Question 4

Have you ever solved for x on an SAT question, but ended up getting the problem wrong because the problem was actually asking for the value of 2x? Most students have had this experience before. This happens becomes we become so engrossed in our calculations that we forget what the original problem was asking for. We are so happy that we finally solved the problem that we forget what the original problem was asking for.

We are so used to our high school math classes being straightforward (i.e. asking for the value of x). However, the SAT is often not as straightforward. In fact, it's downright tricky.

In order to prevent losing sight of the task at hand, you should make it a point to highlight the question. Obviously you will not have a highlighter with you during the SAT. But you can still "highlight" the question by circling it. You should circle the part of the question that asks you exactly what you need to find.

For example, the question might ask for the value of x, $2x$, or x^2. Whatever the question asks you to find should be what you circle. Not only will this clearly indicate what you need to look for as you are solving the problem, but it will also focus your attention.

Many of the SAT Math problems are lengthy word problems. With so much information given, it's easy to get overwhelmed. By highlighting the unknown value of an SAT Math question, your mind can focus on what you need to eventually find.

While highlighting the unknown value is helpful, it won't be much use if you don't look back at what you have circled after you have finished solving a problem. After solving a question, many students often go directly to the answer choices and choose the one that matches what they solved for.

Before going to the answer choices, you should go back to the original question and look at the unknown that you highlighted. Make sure that what you circled in the question is also what you solved for while doing your work. If the two don't agree, you must adjust your answer to find the highlighted unknown.

Although this sounds like a simple principle, it's an important one. If you don't check that you solved for the correct value after every math problem, then you will make a couple of silly mistakes that will cost you big on the SAT Math section.

In addition, think in real world terms. If you are on a student-produced response question that asks about number of pencils, don't fill in 50.3 as your answer. You cannot have 50.3 pencils. Instead, you should fill in 50 or 51 (depending on if the question says to round up or down).

Highlight the question by circling or underlining the unknown so that the worst thing possible doesn't happen to you: getting a question wrong because you forgot what it was originally asking for despite solving the entire problem correctly.

Verify Each Step 5

When do you check your work on an SAT Math section? If you are like most students, you likely check your work at the end after you have finished all problems on the math section.

However, no student in the history of the SAT has ever found a mistake after they have completely finished an SAT Math section. Okay, maybe I'm being a little dramatic. But chances are that you will not find an error after you have completed all of the problems on an SAT Math section.

The reason students do not find errors after they have completed an SAT Math section is that they are typically only "confirming" answers rather than really scrutinizing them to check for mistakes.
Confirming answers is different than looking for errors. When you are done with an SAT Math section, you are likely only reviewing your work.

Rarely do you ever dive deep enough into a problem that you will find an error in the problem. In addition, there is limited time leftover at the end of an SAT Math section. Most students don't have more than 5 minutes left. This causes students to rush even more while they are checking their work. This rush against time results in even fewer errors found at the end of an SAT Math section when most students check their work.

If checking your work at the end of an SAT Math section doesn't work, what does? Checking your work as you work through each problem. As you are solving a problem, you should Verify Each Step. For example, you should verify that 3 x 12 actually does equal 36 while you are solving a problem. You should verify that the formula for the area of a triangle really is ½ bh.

Verifying each step is a much more effective way to check your work because you are fully engrossed in a problem. You will not have to reenter the details of a particular question because you are already knee-deep in those details.

In addition, verifying each step as you go will help prevent frustration. Sometimes a simple arithmetic error can cause your answer to be off despite doing all the correct steps. This can throw you off your game since you know that you are solving the problem correctly. Verifying each step will help you avoid small frustrations like this during the test and ultimately boost your SAT Math score.

Although Verifying Each Step may make you take longer to complete each SAT Math section, you will also be more accurate. You may not have as much time at the end of an SAT Math section as you are used to. However, the time that you used to have at the end of an SAT Math question was useless anyway. This is essentially a story of the tortoise and the hare. Would you rather be the overconfident, fast hare or the slower, more accurate tortoise who still finishes the race?

SAT MATH
EXPERT STRATEGIES

Substitute Abstracts With Tangibles (SAT) 1

Perhaps the most important strategy for the SAT Math section is to Substitute Abstracts with Tangibles (SAT). The primary way to use this strategy is to create your own numbers to plug in for variables in algebraic equations. However, this is not the only use of SAT. You can Substitute Abstract with Tangibles on a variety of other questions including, but not limited to, word problems, geometry questions, and proportions.

The major advantage of Substitute Abstract with Tangibles is that our mind works with tangibles. It is much easier to comprehend that I have 100 cows than it is to comprehend that I have x cows. By taking the abstractness out of problems, we can often solve them in ways that are far easier than traditional high school math classes teach.

One of keys to substituting abstracts with tangibles is to pick the right numbers to substitute. It depends on the problem that you are working on, but I often initially substitute the number 2. I like 2 because it is a small integer that is easy to work with. You want to avoid substituting 0 or 1 because this will often result in multiple answer choices giving you the same answer. Of course, if you ware substituting for an angle measure, then 2 might not be the best number to plug in. Instead, you might want to substitute an angle measure of 60 degrees. Do what makes sense for the problem at hand.

In addition, you want to keep the numbers that you substitute consistent. If you decide to make $x = 2$, then you need to keep $x = 2$ throughout the problem and the answer choices. You cannot decide that $x = 2$, and then make $x = 3$ in one of the answer choices. Furthermore, if y is equal to 50% of x, then y must equal 1. You cannot plug in a different number for y because it is directly related to x, which you have already designated a value for.

Expert Practice

1

A toxic substance has a half-life of 15 years. Half-life is the amount of time it takes for half of the amount of a substance to decay. The amount of the toxic substance at the beginning of 2015 was 200 grams. If P represents the remaining amount of the toxic substance n years after 2015, then which of the following equations represents the right model for the decay of the toxic substance over time?

(A) $P = 200\,(0.5)^{n}$

(B) $P = 200 - 15n$

(C) $P = 200\,(0.5)^{15n}$

(D) $P = 200\,(0.5)^{n/15}$

Solution

1 – Apply SAT

I should substitute a tangible number for the variable n: n = 15 years

→ Notice that it makes sense to substitute 15 (or multiple of 15) since the half-life is 15.

When n = 15, how much of the toxic substance is remaining? Well if we started with 200 grams and 15 years have passed, half of 200 should be remaining, or 100 grams.

Conclusion: When n = 15 years, P = 100 grams

2 – Substitute Abstracts with Tangibles

(A) $P = 200\ (0.5)^{1}$ ✗

(B) $P = 200 - 15(15)$ ✗

(C) $P = 200\ (0.5)^{15(15)}$ ✗

(D) $P = 200\ (0.5)^{15/15}$ ✓

The only answer choice that has P = 100 when n = 15 is answer choice D. Notice how we were able to solve it without using any algebra at all. We did not have to think about how the equation would need to look. Nor did we have to have to think in abstract terms.

If you wanted to solve this problem in the traditional high school math way, you would need to think about how exponential equations are formed. This is a lot to think about under the

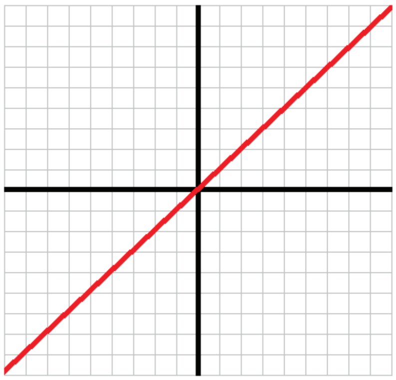
pressure of the SAT. Instead, the tangible numbers we plugged in made this problem extremely simple.

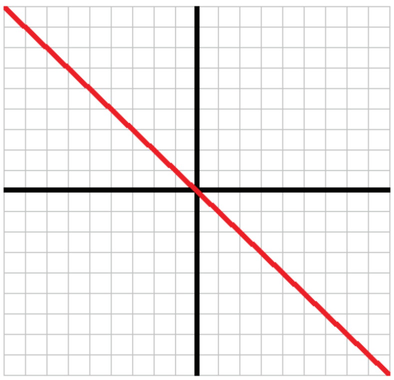
Notice that you did not have to actually solve out each answer choice when you plugged in 15. Instead, you could simply ball park that P would not be equal to 100 in answer choices A – C. For example, $(0.5)^{15*15}$ power clearly is not going to get you to P = 100.

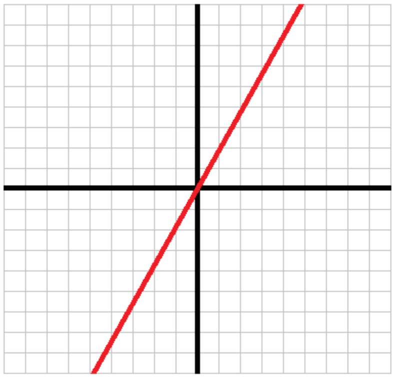
Also notice how you do not need to use a calculator in order to solve this problem (even though this problem would be found on the calculator section of the SAT). In fact, using a calculator to plug in n = 15 into each one of the answer choices would actually take you longer than if you quickly ball parked the answers quickly in your head.

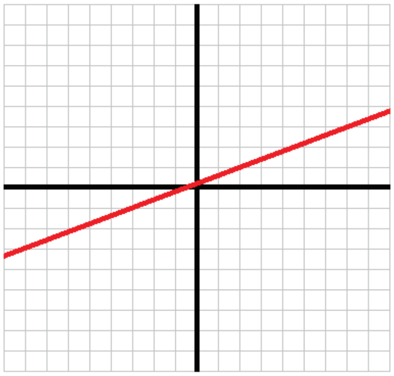
2

If k is a positive constant and less than 1, which of the following could be the graph of $y - x = k\,(x+y)$ in the xy-plane?

(A)


(B)


(C)


(D)


Solution

1 – Apply SAT

Although this may not look like a problem that you would classically use the Expert Strategy SAT on, my mind immediately thinks of this strategy when I see statement such as "k is a positive constant and less than 1." This means we should pick a simple tangible number to substitute for the abstract variable k that satisfies the requirements of the problem: $k = \frac{1}{2}$

2 – We should rearrange the equation in the problem into the slope-intercept form.

$$y = mx + b$$

Whenever you see the equation for a linear graph on the SAT, you should rearrange it into the

y = mx + b form. The m-value indicates the slope of the graph and the b-value indicates the y-intercept of the graph.

$$y - x = \frac{1}{2}(x + y)$$
$$y - x = \frac{1}{2}x + \frac{1}{2}y$$
$$y = 1.5x + \frac{1}{2}y$$
$$0.5y = 1.5x$$
$$y = 3x$$

3 – Substitute Abstracts with Tangibles

Now that we have rearranged the linear graph equation into the y = mx + b form, we can determine the slope and the y-intercept.

slope = 3

y-intercept = 0

Therefore, we know that when $k = \frac{1}{2}$ **, the slope of the graph should be 3, and the y-intercept should be 0.**

Let's evaluate the answer choices.

(A) Slope of 1 and y-intercept of 0.

(B) Slope of –1 and y-intercept of 0.

(C) Slope of $\frac{3}{2}$ and y-intercept of 0.

(D) Slope of $\frac{1}{3}$ and y-intercept of 0.

Although none of the answers have a slope of 3, we can still choose C as our answer. It is the only one that has a slope greater than 1 (similar to 3).

I was able to quickly calculate the slopes of each of the given graphs by simply counting rise over run. However, it was not necessary to calculate the slopes of all of the graphs individually. Once again, you could have simply ball parked that answer choice C had the steepest slope (the one that is closest to 3); therefore, C is the answer.

Notice how this wasn't a purely algebraic problem. Instead, this graphing problem was solved using Substitute Abstract with Tangibles. We were able to avoid having to do a lot of abstract thinking because of the Math Expert Strategy of SAT.

We did not have to think about, "Well, if k is a fraction, then the graph's slope would be the inverse of that fraction, which would then result in a slope that is greater than 1, etc." You certainly could have done it this way (especially if you are astute at thinking about abstract concepts). But under the pressure of the SAT, it is often safer to simply Substitute Abstract with Tangibles so that you don't make a silly mistake.

Substitute Answers in Problem (SAP) 2

This is essentially the sister strategy to Substituting Abstracts with Tangibles (SAT). With

Math Expert
Strategy #1 SAT, we used our own numbers that we created to plug into the variables or unknowns of a question. With Math Expert Strategy #2 SAP, we plug the answer choices given to us back into the original question. Essentially, we are no longer just randomly choosing our own numbers to substitute.

SAT and SAP are not only similar acronyms, but they are also very similar strategies. Try not to get them confused. But even if you do not know which strategy you are using, you can still have success on the SAT Math section. The important thing is just to use both strategies as much as possible.

How do you know when to use SAT and when to use SAP? Look at the answer choices. If there are variables in the question and variables in the answer choices, then use Math Expert Strategy #1 SAT. If there are variables in the question and tangible numbers in the answer choices, then use Math Expert Strategy #2 SAP. Of course, you won't be able to use these two strategies on every question that contains variables, but you should start by checking to see if you can use these strategies before trying traditional high school math.

Substituting Answers in Problem is extremely unintuitive. High school math classes typically do not give us multiple-choice exams. Why would a math teacher not give a multiple-choice algebra test?
Because you could plug the answer choices into the original problem. Essentially, you could skip having to do any algebra at all.

But the SAT actually does do this! The exam often gives you algebra questions (and other questions involving variables) with multiple-choice answers. We can take advantage of the multiple-choice nature of the test by simply plugging the answer choices back into the original problem.

However, our minds are not accustomed to think in this way because high school math classes don't teach us to work bottom-up (instead they program our minds to think top-down).

Expert Practice

1

The "Star Factory" creates pens. The cost C of creating n pens is given by the equation $C = 4n + 120$. The company makes a profit when total income from selling a quantity of pens is greater than the total cost of producing that quantity of pens. The selling price of a pen is \$10. Which of the following inequalities gives all possible values of n for which the company will make a profit?

(A) $n < 20$

(B) $n > 20$

(C) $n < 40$

(D) $n > 40$

Solution

1 – Apply SAP

Instead of diving right into the problem and writing out algebraic equations like most students would do, let's really examine the answer choices. We can immediately elim-

inate answer choices A and C because I know that as I make more pens, the more money I will make. Answer choices A and C say that as fewer pens are created, we make greater profits. That is simply not true.

→ Let's then use SAP on answer choice B.

n = 21

Answer choice B states that n > 20. Therefore, I should substitute 21 into the problem.

→ Calculate Cost

The cost of creating 21 pens is C = 4(21) + 120 = $204

→ Calculate Revenue

The revenue generated by selling 21 pens is 21 x $10 = $10

→ Calculate Profit

$210 – $204 = $6

Does the company make a profit when it sells 21 pens? Yes. Therefore, our answer is B. You may be wondering why the answer is not D. Because answer choice D does not include when the company makes a profit when it sells 21 pens, 22 pens, 23 pens, etc. Notice how we solved this problem without having to use any algebra! That is the power of SAP.

Note: You could have certainly solved this problem using traditional high school algebra.

However, I am simply showing you how to apply SAP to solve the problem in a way that you are probably not used to.

2

The function f is defined by $f(x) = 2x^3 - 9x^2 + cx - 6$ where c is a constant. In the xy-plane, the graph of f intersects the x-axis at three points $(-1,0)$, $\left(-\frac{1}{2},0\right)$, and $(p,0)$. What is the value of c ?

(A) -17

(B) -3

(C) 3

(D) 17

Solution

1 – Apply SAP

Instead of diving right into the problem and writing out algebraic equations like most students would do, let's use SAP. SAP works a little differently on graph questions. Whenever you have points of a graph and the equation of a graph, the "answers" that you plug into the equation are not in the answer choices. Instead, the "answers" are the coordinates that are given.

Therefore, in order to solve this question we should simply choose one of the given coordinates and plug it into the equation. Coordinates are written in the form (x, y). And f(x) is essentially the same thing as y. Therefore, we have everything we need to plug in.

Plug the coordinate (–1, 0) into the equation.

$$f(x) = 2x^3 - 9x^2 + cx - 6$$
$$\rightarrow 0 = 2(-1)^3 - 9(-1)^2 + c(-1) - 6$$
$$\rightarrow 0 = -2 + -9 + -c + -6$$
$$\rightarrow 0 = -17 + -c$$
$$\rightarrow c = -17$$

* Note: Whenever I have a subtraction sign in an equation, I change it to the addition of a negative number. This helps me keep the signs of different numbers straight. Try it out!

The answer is A. Notice how we solved this problem without having to use any algebra or think in abstract terms! In fact, we didn't even have to figure out all of the zeros of the equation or what p is.

Notice that there is a broad definition of "answers" in Substitute Answers in Problem. In this case, the answers that we plugged in were not necessarily the answer choices, but the answers to the graph via the coordinates given.

Therefore, you should be able to adjust each strategy for the problem given. Do not get too stuck in your ways. After you have practiced strategies such as SAT and SAP enough, you will eventually be able to apply them in a variety of different ways. You will also eventually be thinking of SAT and SAP first, before thinking of traditional high school math – I certainly do!

Just Get Started 3

This strategy is more important than ever on the SAT. The New SAT includes longer word problems on the math section than any previous version of the SAT. So you have to be on your A game when completing the SAT Math section.

The problem many students face when dealing with long word problems is that they get lost in the details. If a problem throws 6 different variables, 2 equations, and a bunch of other details at you, it's easy to get confused. When students get confused by looking at so much information at once, they often give up on the problem before even starting.

To avoid confusion on the SAT Math section, Just Get Started. Just Get Started means that you should start with the first step of any problem. If you can simplify an equation, simplify the equation. If you can divide two numbers, divide the numbers. If you can draw a diagram, draw a diagram.

While this may sound like a very simple technique, it's very powerful. Many students don't get the problem started because of how we have been trained to solve math problems in high school. In high school, math teachers tell us exactly how to solve a problem step-by-step, from A to Z.

However, on the SAT Math section, you may not be able to visualize exactly how to solve a problem from start to finish. This causes many students to get stage fright. If they don't know exactly how to solve a problem, they don't start the problem at all. Do not wait for the solution to come to you. The solution will not magically appear if you stare long enough at the problem. You have to go to the solution by getting the problem started.

Just Get Started.

Even if you have no idea how to solve a problem, just do the first step that you can. Not knowing what the next step is can be scary. But if you simply Put Your Pencil to Paper and Just Get Started, you will surprise yourself. Suddenly, after you complete the first step, the second step will be clear, then the next step will be clear, and so on.

However, don't let this strategy distract you from looking at the big picture. For example, look at the following problem:

$$4\,(2x - 2.2) = 3\,(2x - 2.2)$$

If you Just Get Started, you may miss the fact that multiplying the same expression by 4 and 3 can only yield the same answer if $2x - 2.2 = 0$. So $x = 1.1$

Although Just Get Started is an extremely powerful strategy, do not let it distract you from the looking at the bigger picture in problems…which can often lead to a quick solution!

Expert Practice

1

In the equation below, what is the value of x ?

$$\frac{x-3}{2}-\frac{1+x}{3}=1-\frac{x}{6}$$

(A) $\frac{2}{17}$

(B) 2

(C) $\frac{17}{2}$

(D) 17

Solution

1 – Just Get Started

In order to illustrate this Math Expert Strategy, we have to pretend that we have no idea how to solve this problem. Let's pretend that we have no idea what to do in the above question.

However, what we do notice is that there is a common denominator of 6 among all of the

fractions. I should Just Get Started by getting all of the fractions to have a denominator of 6.

$$\frac{3(x-3)}{6}-\frac{2(1+x)}{6}=\frac{6}{6}-\frac{x}{6}$$

2 – First Step Leads to Next Steps

$$\frac{3x-9}{6}-\frac{2+2x}{6}=\frac{6-x}{6}$$

$$3x+\ -9+\ -2+\ -2x=6-x$$

$$x+\ -11=6-x$$

$$2x=17$$

$$x=\frac{17}{2}$$

3 – Select Answer

Select answer choice C. Notice how much less intimidating the problem got after we Just Got Started. Once all of the fractions had the same denominator that we could cancel, it became much easier than staring at a problem with all kinds of different denominators and expressions. The first step led to the second step, which led to the third step, etc. Just Got Started to solve questions you don't know how to solve on the SAT Math section!

Ditch The Tech 4

When I watch students take SAT exams, I can often tell who is going to score poorly on the SAT Math section. I don't need to know anything about the student. It also has nothing to do with his or her facial expressions during the SAT Math section.

There is a simple way to tell which students will score poorly on the SAT Math section. Students who go through the SAT Math section with their calculator in their hands are going to score poorly.

I'm not referring to students who occasionally pick up their calculator to do a calculation. But students who have the calculator in their hands for the majority of the SAT Math section because they are trying to solve questions in the calculator will score poorly.

The calculator will not save you. The best way to solve SAT Math problems is with your pencil, not your calculator. Working problems out with your pencil glued to the test booklet is when the real
thinking happens. If you try to solve problems in your calculator, very little thinking happens. You are trying to rely on your calculator to do the thinking for you, which won't work.

Remember that thinking power and working memory are inversely related. If you are trying to solve problems in your calculator, then you are also attempting to remember many things in your head. This will take up working memory and reduce your thinking power. Don't let the calculator handicap you on the SAT Math section and Ditch the Tech!

This does not mean you shouldn't use the calculator at all on the SAT Math section. In fact, sometimes it is advantageous to use the calculator. But you should limit your calculator use. Take problems as far as you can without a calculator. Then, after you have solved as much of the problem as you can with your pencil, then use your calculator.

For example, you may have setup an entire equation by hand first. Then at the end of the problem, you should plug in the final numbers to get the exact value with your calculator. You should not be attempting to solve problems from the beginning using your calculator.

In addition, rounding off large numbers can be especially useful. It may not be necessary to do the following calculation: 458,234,239 – 134,234,245. Instead, simply doing 458 – 134 = 324 may be good enough. Chances are, there is only one answer choice in the 324,000,000 range.

Expert Practice

1

A manager of a company predicted that the profit of the company will double every 4 years. The profit at the beginning of 2014 was estimated to be $200,000.
If P represents the profit n years after 2014, then which of the following equations represents the manager's model of the profit over time?

(A) $P = 200{,}000\ (2)^{4n}$

(B) $P = 200{,}000\ (2)^{n/4}$

(C) $P = 200{,}000\ (2)^{4/n}$

(D) $P = 200{,}000\ (2)^{n+4}$

Solution

1 – Ditch The Tech

On the surface, this may look like a problem that requires a calculator. The answer choices look complex. Most students would jump immediately to their calculators. However, let's Ditch the Tech and think about this problem logically.

2 – Substitute Abstract with Tangibles

Because there are variables in the question and variables in the answer choices, I can likely use Math Expert Strategy #1: Substitute Abstract with Tangibles. A strategic tangible number to plug in for the variable n would be a multiple of four:

$n = 8$

What will the profit be in 8 years if the profit doubles every 4 years?

In 4 years, the profit will be $400,000

In 8 years, the profit will be $800,000

3 – Substitute Tangibles into Answer Choices

(A) $P = 200{,}000\,(2)^{4*8}$ ×

(B) $P = 200{,}000\,(2)^{8/4}$ ✓

(C) $P = 200{,}000\,(2)^{4/8}$ ×

(D) $P = 200{,}000\,(2)^{8+4}$ ×

Ditch the Tech. Although most students believe that they need to calculate the exact values of the above plug-ins, that is not true. Instead, you can simply ballpark the correct answer.

Answer choice (A) is not correct because it will be much larger than \$800,000. Answer choice (B) is correct because \$200,000 x 4 is exactly equal to \$800,000. Answer choice (C) is incorrect because \$200,000 x $\sqrt{2}$ is much smaller than \$800,000. Answer choice (D) is incorrect because it will be much larger than \$800,000.

Notice how we did not have to use a calculator on this problem despite working with very large numbers. Ditch the Tech will often help you solve problems more efficiently!

2

Peter buys the same present from an online shop x times . A tax of 9% is applied to the present's initial price, and the seller charges an additional one-time, untaxed shipping fee of \$7 to ship one box containing all of the presents. Peter's credit card was charged a total of T dollars. Which of the following represents the present's initial price in dollars?

(A) $\frac{(T-7)x}{1.09}$

(B) $\frac{T}{1.09x}-7$

(C) $\frac{T-7}{1.09x}$

(D) $\frac{(T-7)(1.09)}{x}$

Solution

1 – Ditch The Tech

On the surface, this may look like a problem that requires a calculator. The answer choices look complex. Most students would jump immediately to their calculators. However, let's Ditch the Tech and think about this problem logically.

2 – Substitute Abstract with Tangibles

Because there are variables in the question and variables in the answer choices, I can likely use Math Expert Strategy #1: Substitute Abstract w/ Tangibles. Let's plug in the following numbers:

$x = 2$

$T = \$28.80$

* I strategically plugged in $28.80 because I'm assuming the price of a present is $10. If you add a 9% tax to $10, then the price of each present is $10.90. If you buy two presents, the cost would be $21.80. Add a shipping fee of $7 and the total cost is $28.80.

Now we need to find an answer choice that gives us a $10 original price when we plug in $x = 2$ and $T = \$28.80$

3 – Substitute Tangibles into Answer Choices

(A) $\frac{(28.80-7)(2)}{1.09}$ ×

(B) $\frac{28.80}{1.09(2)}-7$ ×

(C) $\frac{28.80-7}{1.09(2)}$ ✓

(D) $\frac{(28.80-7)(1.09)}{2}$ ×

Once again, you can Ditch the Tech. Although most students would believe that they need to calculate the exact values of the above plug-ins, that is not true. You can simply ball-park the correct answer.

Answer choice (A) is incorrect because it will create an answer that is approximately $\frac{42}{1.09}$ **, which is not $10. Answer choice (B) is incorrect because it will create an answer that is approximately 14 – 7, which is not $10. Answer choice (C) is correct because it will create an answer that is approximately** $\frac{21.80}{2.18}$ **, which is $10. Answer choice (D) is incorrect because it will create an answer that is approximately** $\boxed{\frac{42}{2}}$**, which is not $10. Notice how we did not have to use a calculator on this problem despite working with numbers filled with decimals. Ditch the Tech will often help you solve problems more efficiently!**

Sketch a Diagram 5

SAT Math problems will often describe a geometric shape, a graph, a series of items, etc.

However, the items described will not always be drawn out for you. I am amazed by the number of students who don't take the time to draw out a diagram themselves. Visualizing a diagram, especially on word problems, can be powerful on the SAT Math section. Sketch a Diagram whenever one is not given to you on the SAT Math section.

The sister Math Expert Principle to this strategy is Label Everything. Make sure that you label every aspect of your diagram. A poorly labeled diagram may be worse than no diagram at all. Labeling a diagram is important because it will help you clearly visualize the problem.

Along the same lines, make sure that your diagrams are large. I personally have gotten questions wrong in the past because my diagrams were too small and I couldn't really see the diagram correctly. Draw your diagrams to scale as best as you can!

Sketching a Diagram also helps with Putting Pencil to Paper. The more that you draw diagrams, the more you are thinking with your pencil. The more that you think you with your pencil, the higher your SAT score will go!

Expert Practice

1

A pool owner is draining his pool. 1.5 hours after the drain started, he measures the remaining water in the pool and finds that it is 80 cubic meters. As the drain continues, he takes another note 2 hours and 45 minutes after the draining process started and finds that the remaining water in the pool is now 75 cubic meters. Provided that the volume of the water in the pool decreases at a constant rate as time elapses, which of the following linear models best describes the remaining volume v of the water in the pool in cubic meters at t hours since the draining process started?

(A) $v = -\frac{5}{.95}t + 80$

(B) $v = \frac{5}{.95}t + 86$

(C) $v = -4t + 80$

(D) $v = -4t + 86$

Solution

1 – Sketch a Diagram

On the surface, this may not look like a problem that requires a diagram. But we should quickly Sketch a Diagram to assure that we understand the problem.

Time 0-------------------1.5h----------------2.75h

Water Level --------------------80m³--------------75m³

Notice how I didn't sketch out a picture of the actual pool. Instead, I simply drew a timeline and the corresponding water levels at various elapsed times given by the problem (note that 45
minutes is equivalent to 0.75 hours). Your diagram only needs to make sense to you and does not have to necessarily be a picture.

2 – Substitute Abstract with Tangibles

Because there are variables in the question and variables in the answer choices, I can likely use Math Expert Strategy #1: Substitute Abstract with Tangibles. A strategic tangible number to plug in for the variable t would be an elapsed time that is already given to us:

t = 1.5

What is the remaining volume in the pool after 1.5 hours have elapsed (it's actually already given in the problem)?

v = 80m³

3 – Substitute Tangibles into Answer Choices

(A) $v = -\frac{5}{.95}(1.5) + 80$ ✗

(B) $v = \frac{5}{.95}(1.5) + 86$ ✗

(C) $v = -4(1.5) + 80$ ✗

(D) $v = -4(1.5) + 86$ ✓

You do not need a calculator to find the exact values above. You should be able to quickly ballpark the answer.

Answer choice (A) is incorrect because it will create an answer that is approximately 70, which is not 80.

Answer choice (B) is incorrect because it will create an answer that is approximately 94, which is not 80.

Answer choice (C) is incorrect because it will create an answer that is 74, which is not 80.

Answer choice (D) is correct because it will create an answer that is 80. Notice that you did not need to Sketch a Diagram to solve this question. However, the diagram can often help you grasp what is going on in a word problem such as this one.

2

Mia is on a diet. She loses 2 pounds every week. After 3 weeks she weighs 128 pounds. Which of the following models best describes the weight w after t weeks?

(A) $w = 2t + 128$

(B) $w = -2t + 134$

(C) $w = 2t + 134$

(D) $w = -2t + 128$

Solution

1 – Sketch a Diagram

On the surface, this may not look like a problem that requires a diagram. However, we should quickly Sketch a Diagram to assure that we understand the problem.

Time 0---------1w-----------2w------------------3w

Weight 134------132---------130-----------------128

Again, I simply drew a timeline and the corresponding weights. I was able to calculate Mia's weight each week by simply adding back the two pounds the problem stated that she lost each week.

2 – Substitute Abstract with Tangibles

Because there are variables in the question and variables in the answer choices, I can likely use Math Expert Strategy #1: Substitute Abstract w/ Tangibles.

A strategic tangible number to plug in for the variable t would be an elapsed time that is already given to us:

$t = 3$

What is Mia's weight after 3 weeks?

$w = 128$

3 – Substitute Tangibles into Answer Choices

(A) w = 2(3) + 128 ×

(B) w = –2(3) + 134 ✓

(C) w = 2(3) + 134 ×

(D) w = –2(3) + 128 ×

You do not need a calculator to find the exact values above. You should be able to quickly ballpark the answer.

Answer choice (A) is incorrect because it will create an answer that is 134, which is not the 128 we are looking for.

Answer choice (B) is correct because it will create an answer that is the 128 we are looking for.

Answer choice (C) is incorrect because it will create an answer that is 140, which is not the 128 we are looking for.

Answer choice (D) is incorrect because it will create an answer that is 122, which is not the 128 we are looking for.

Notice that you did not need to Sketch a Diagram to solve this question. However, the diagram can often help you grasp what is going on in a word problem such as this one.

Be sure to Sketch a Diagram frequently on the SAT Math section!

Ace Equations 6

Simple Equation

You should be able to solve a simple equation.

$$\frac{1}{3}x + 11 = 20$$

$$\frac{1}{3}x = 9$$

$$x = 27$$

System of Equations

You should be able to solve a system of equations. One of the ways that the College Board test question writers have reduced the use of Substituting Abstract with Tangibles is by incorporating systems of equations into the New SAT. You can typically not use Substitute Abstract with Tangibles on questions that have systems of equations. So these questions will often take a little longer to do.

$$\begin{cases} 2x - y = 2y - 5 \\ x + 5y = 3 \end{cases}$$

$$x + 5y = 3$$

$$x = 3 - 5y$$

$$2(3 - 5y) - y = 2y - 5$$

$$(6 - 11y) = 2y - 5$$

$$11 = 13y$$

$$y = \frac{11}{13}$$

$$x = 3 - 5y$$

$$x = 3 - 5\left(\frac{11}{13}\right)$$

$$x = -\frac{16}{13}$$

No Solution System of Equations

You should be able to recognize when a system of linear equations has no solutions. Essentially, the slopes of both lines are the same because the lines are parallel and do not intersect. If the coefficients on x and y are the same and they have a different y-intercept, then you are dealing with parallel lines with no solution. If the coefficients on x and y are the same and they have the same y-intercept, then you are dealing with the same line with infinite solutions.

$$\begin{cases} \frac{1}{4}x - \frac{1}{12}y = 7 \\ 12x - 4y = 10 \end{cases}$$

$$48\left[\frac{1}{4}x - \frac{1}{12}y = 7\right]$$

$$12x - 4y = 336$$

$$12x - 4y = 10$$

No Solution

Manipulate Equations

You should be able to manipulate equations.

If $\frac{1}{8}x+\frac{1}{4}y=10$**, what is the value of** $2x+4y$ **?**

$$\frac{1}{8}x+\frac{1}{4}y=10$$

$$\frac{16}{8}x+\frac{16}{4}y=10(16)$$

$$2x+4y=160$$

Factor Quadratic Equations

A quadratic is an expression or equation that has a single squared term as its highest power. You should be able to factor quadratic equations.

$$x^2-3x=70$$

$$x^2-3x-70=0$$

$$(x-10)(x+7)=0$$

Zeros of Quadratic: 10 and –7

→ **You should also be able to recognize perfect square quadratics**

$$x^2+4x+4=0$$

$$(x+2)^2=0$$

→ **You should also be able to recognize classic quadratics:** $x^2 - y^2 = (x+y)(x-y)$

$$9x^2 - 25 = 0$$
$$(3x+5)(3x-5) = 0$$

→ **You should also be able to factor higher power equations**

$$9x^6 - 25x^4 = 0$$
$$(3x^3 + 5x^2)(3x^3 - 5x^2) = 0$$

Quadratic Formula

You should be able to solve for x using the quadratic formula.

$$x^2 - 3x = 70 \Leftrightarrow x^2 - 3x - 70 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{3 \pm \sqrt{-3^2 - 4(1)(-70)}}{2(1)}$$
$$x = \frac{3 \pm \sqrt{289}}{2(1)}$$
$$x = \frac{3 \pm 17}{2(1)}$$
$$x = 10 \quad or \quad -7$$

→ **You should also note, the sum of the solutions is equal to –b and the product is equal to c.**

For example, 10 + –7 = 3 and 10 * –7 = –70.

Discriminant of the Quadratic Formula

The discriminant of the quadratic formula is the part under the square root sign: $b^2 - 4ac$

Positive Discriminant → 2 Solutions (or Roots/Zeros)

Negative Discriminant → No Solutions (or Roots/Zeros)

Discriminant = 0 → 1 Solution (or Roots/Zeros)

For example, let's say that a system of equations intersects when $x^2 - 8x - 8 = 0$ **.**
We can quickly determine how many times the graphs intersect simply by examining the discriminant.

$$b^2 - 4ac$$

$$(-8)^2 - 4(1)(8)$$

$$32$$

Positive Discriminant = 2 Solutions = two graphs in the system of equations intersect twice!

Treating Expressions As Variables

You should be able to notice when an expression can be treated as a variable.

$$(x-2)^2 + 6(x-2) + 8 = 0$$

In this case, you should treat the expression $(x-2)$ **just as you would "x."**

$$[(x-2)+4][(x-2)+2] = 0$$

$$[(x-2)+4] = 0 \ or \ [(x-2)+2] = 0$$

$$x = -2 \ or \ x = 0$$

Absolute Value

Absolute value expressions can never be equal to a negative number.

→ $|x+1|+2 \neq 0$

The absolute value difference between two numbers is the distance between those two numbers on the number line.

→ $|-2-4|=6$

Convert Word Problems to Equations

You should be able to read a word problem and translate it into an equation.

The cost of a banana is $1 and an apple is $2. You purchased a total of $100 in apples and bananas. The total number of apples and bananas is 50. Setup a system of equations.

$$a + b = 50$$

$$b + 2a = 100$$

where: a = # of apples

b = # of bananas

Expert Practice

1

If $a^2 + 15a = 76$ and $a > 0$, what is the value of $a + 5$?

Solution

1 – Ace Equations

This is a free response question so we don't have answer choices A – D to help us. Nevertheless, we know how to Ace Equations. First, set the equation equal to 0.

$$a + 15a - 76 = 0$$

Next, try to think of two numbers that equal 76 when multiplied together.

2 & 38

Unfortunately, there is no way for me to add or subtract 2 and 38 in order to get 15. Therefore,

I should try to think of two different numbers that equal 76 when multiplied together.

4 & 19

This makes sense. When I subtract 4 from 19, I get 15. I can now factor the equation.

$$(a + 19)(a - 4) = 0$$

$$a = 4 \ \ or \ \ -19$$

2 – Solve Unknown

Filling in a = -19 is not correct. Remember to Highlight the Question. The unknown in the problem was actually “a + 5”. In addition, the problem states that a > 0. Therefore, a = 4 and a + 5 = 9. We can grid-in 9 as our answer.

2

Based on the system of equations below, what is the value of $x + y$?

$$\begin{cases} 4x-5-3y=2x+4 \\ x+5y+1=y \end{cases}$$

(A) −4

(B) −2

(C) 2

(D) 4

Solution

1 – Ace Equations

The first step should be to get all of the variables to one side.

$$4x-5-3y=2x+4$$

$$2x-5=3y+4$$

$$2x=3y+9$$

$$x=\frac{3}{2}y+\frac{9}{2}$$

Next, let's get all of the variables on the same side in the second equation.

$$x + 5y + 1 = y$$
$$x = -4y - 1$$

Now, let's plug in the x-value of the first equation into the second equation.

$$\frac{3}{2}y + \frac{9}{2} = -4y - 1$$
$$\frac{3}{2}y + \frac{11}{2} = -4y$$
$$\frac{11}{2}y + \frac{11}{2} = 0$$
$$\frac{11}{2}y = -\frac{11}{2}$$
$$y = -1$$

Now, let's plug in the y-value into the equation that yields a value for x.

$$x = -4(-1) - 1$$
$$x = 4 - 1$$
$$x = 3$$

2 – Solve Unknown

Finally, let's solve for what the original question was asking us for: x + y.

This is just 3 + –1.

Therefore our answer is 2. Select answer choice C.

7

Ace Expressions

Multiply by 1

When adding or subtracting fractions that have different denominators, you can multiply each fraction by 1 to get a common denominator.

$$\frac{5x}{x+2}-\frac{10}{x-2}$$

$$\frac{5x(x-2)}{x+2(x-2)}-\frac{10(x+2)}{x-2(x+2)}$$

$$\frac{5x(x-2)-10(x+2)}{(x+2)(x-2)}$$

$$\frac{(5x^2-10x)-(10x+20)}{(x+2)(x-2)}$$

$$\frac{5x^2-20x+20}{(x+2)(x-2)}$$

Multiply the Negative Through

Sign errors are one of the most common errors students make when dealing with expressions. I often make them myself, especially when I'm working too fast. I have found two effective ways to reduce sign errors:

- Anytime there is subtraction in an expression, change it to the addition of a negative
- Anytime an expression has multiplication of a negative, multiply and distribute that negative through

$$2x-3(4x+5)$$

$$2x+\ -3(4x+5)$$

$$2x+\ -12x+\ -15$$

$$-10x+\ -15$$

Fractional Expressions - Separation

When there is an expression in the numerator of a fraction, you should be able to separate it out into it component parts.

$$\frac{2x+3x}{x-2}$$

$$\frac{2x}{x-2}+\frac{3x}{x-2}$$

Fractional Expressions - Rationalization

When there is an expression in the denominator of a fraction that needs rationalization, you multiply by the fraction by the denominator's conjugate. The conjugate is the exact same expression, but with the opposite sign in between.

$$\frac{2x+3x}{x-\sqrt{2}}$$

$$\frac{2x+3x}{x-\sqrt{2}}\left(\frac{x+\sqrt{2}}{x+\sqrt{2}}\right)$$

$$\frac{2x^2+2x\sqrt{2}+3x^2+3x\sqrt{2}}{x^2+2}$$

$$\frac{5x^2 \quad 5x\sqrt{2}}{x^2 \quad 2}$$

Fractional Expressions – Clearing Variable Denominators

To solve an equation with fractions with different expressions in the denominators, multiply both sides by both expressions.

$$\frac{3}{x-5}=\frac{7}{x+3}$$

$$(x+3)(x-5)\frac{3}{x-5}=\frac{7}{x+3}(x+3)(x-5)$$

$$(x+3)3=7(x-5)$$

$$9x+9=7x-35$$

$$2x=-44$$

$$x=-22$$

Recognize Perfect Squares

You should be able to recognize perfect squares.

$$4x^2 - 20x + 25$$

$$(2x-5)(2x-5)$$

$$(2x-5)^2$$

Substitute Expressions

You should be able to substitute one expression for another.

If $y = x^3 - 5x + 2$ and $z = x^2 + 10x - 6$, what is $z + 2y$?

$$z + 2y$$

$$x^2 + 10x - 6 + 2(x^3 - 5x + 2)$$

$$x^2 + 10x - 6 + 2x^3 - 10x + 4$$

$$2x^3 + x^2 - 2$$

Adding/Subtracting Polynomials

A polynomial is an expression with at least two algebraic terms, often to different powers.

You should be able to add and subtract polynomials.

$$(x^3y^2 + 6y^3 - 10x^2y^3) - (2x^3y^2 + 6x^2y^3 + 6y^3)$$

$$(x^3y^2 + -10x^2y^3) + (-2x^3y^2 + -6x^2y^3)$$

$$-x^3y^2 + -16x^2y^3$$

Multiplying/Dividing Polynomials

You should be able to multiply polynomials.

$$(x^3y^2 + 10x^2y^3)(2x^3y^2 - 6y^3)$$

$$2x^6y^4 - 6x^3y^5 + 20x^5y^5 - 60x^2y^6$$

You should be able to divide polynomials.

$$x^2 + 3x + 5 \div x + 1$$

$$\begin{array}{r|l} & \quad\quad\;\; x+2 \\ x+1 & x^2+3x+5 \\ & \underline{-x^2 - \;\; x} \\ & \quad\quad\; 2x+5 \\ & \quad\quad\; \underline{-2x-2} \\ & \quad\quad\quad\quad 3 \end{array}$$

Solution: $(x+2) + \dfrac{3}{x+1}$

You should be able to divide polynomials in unconventional ways.

$$x^4 + 5x^3 + bxy + 3y$$

If the polynomial is divisible by $x + 5$, what is the value of b (where b is a constant)?

Factor out x³ from the first part of the equation: $x^3(x+5) + bxy + 3y$

$$bxy + 3y = (x+5)(by)$$

Therefore, bxy + 3y needs to also be divisible by x + 5: $bxy + 3y = bxy + 5by$

$$3y = 5by$$

$$3 = 5b$$

And so, b = $\dfrac{3}{5}$

You should be able to manipulate polynomials.

$$f(x) = 3x^3 + 9x^2 + 3x$$
$$g(x) = x^2 + 3x + 1$$

Which of the following polynomials is divisible by $3x - 1$?

(A) $h(x) = f(x) + g(x)$

(B) $k(x) = f(x) - g(x)$

(C) $m(x) = 3f(x) - g(x)$

(D) $n(x) = f(x) + 3g(x)$

Solution

Factor f(x): $f(x) = 3x(x^2 + 3x + 1)$

Substitute g(x): $f(x) = 3x \cdot g(x)$

Put answer choices in terms of g(x):
(A) $h(x) = 3x \cdot g(x) + g(x)$
(B) $k(x) = 3x \cdot g(x) - g(x)$
(C) $m(x) = 9x \cdot g(x) - g(x)$
(D) $n(x) = 3x \cdot g(x) + 3g(x)$

Factor out g(x):
(A) $h(x) = g(x)(3x + 1)$
(B) $k(x) = g(x)(3x - 1)$
(C) $m(x) = g(x)(9x - 1)$
(D) $n(x) = g(x)(3x + 3)$

Clearly only answer choice B is divisible by 3x + 1

Abstract Polynomials

You should be able to create a polynomial when not much information is given.

Create a polynomial $p(x)$ for which $p(5)$ is equal to 2.

Start by thinking of what number you would have to subtract from x (which is 5 in this case) in order to get 2.

x – 3

Next think of what number you would have to subtract from x in order to get 1.

x – 4

2 multiplied by 1 is equal to 2. So multiplying these two expressions together should give you a polynomial for which p(5) is equal to 2.

$$(x-3)(x-4)$$

$$x^2-4x-3x+12$$

$$x^2-7x+12$$

Note that there are multiple polynomials for which p(5) is equal to 2. The above polynomial is just one that works.

Roots/Zeros/x-intercepts of Polynomials

A root or zero of a polynomial is the x-value that makes a polynomial equal to zero.

$$x^2 - 7x + 12$$
$$(x-3)(x-4)$$

Roots/Zeros/x-intercepts: x = 3 and x = 4

Factors of Polynomials

A factor of a polynomial is an expression that divides evenly into the polynomial.

$$x^2 - 7x + 12$$
$$(x-3)(x-4)$$

Factors: (x – 3) and (x – 4)

Can you think of what expression would be a factor of the polynomial f(x) by looking at the table

below?

x	f(x)
1	10
2	6
3	0
4	–2

Because f(x) is 0 when x =3, the expression (x – 3) would divide evenly and is a factor of f(x)

Expert Practice

1

If the expression $\dfrac{5-9x^2}{2-3x}$ is written in the equivalent form $\dfrac{1}{2-3}+$, what is A in terms of x ?

(A) $2 - 3x$

(B) $2 + 3x$

(C) $9x^2$

(D) 5

Solution

1 – Ace Expressions

Remember that in order to add or subtract numbers with different denominators, you need

to multiply by 1. So first, multiply A by 1 (using the denominator of the other term in the expression).

$$\frac{1}{2-3x}+A$$

$$\frac{1}{2-3x}+A\left(\frac{2-3x}{2-3x}\right)$$

$$\frac{1-2A+\ -3xA}{2-3x}$$

Set both expressions equal to each other.

$$\frac{1+2A+\ -3xA}{2-3x}=\frac{5-9x^2}{2-3x}$$

Since both sides of the equation have the same denominator, we can cancel the denominators out.

$$1+2A+\ -3xA=5-9x^2$$

Solve for A.

$$2A+\ -3xA=4-9x^2$$

$$A(2+\ -3x)=4-9x^2$$

$$A=\frac{4-9x^2}{2-3x}$$

Factor the perfect square in the numerator.

$$A=\frac{(2-3x)(2+3x)}{2-3x}$$

$$A=2+3x$$

2 – Solve Unknown

Select answer choice B.

Note: You can also solve this problem using Substitute Abstract with Tangibles. Try it out by plugging in x = 1 into the first expression.

2

If $y = x^3 + 2x^2 + 1$ and $z = x^2 - 5x + 3$, what is $y - 2z$ in terms of x ?

(A) $x^3 - 4x^2 - 10x - 5$

(B) $x^3 - 4x^2 + 10x - 5$

(C) $x^3 - 10x - 5$

(D) $x^3 + 10x - 5$

Solution

1 – Ace Expressions

Begin with the expression that you need to find.

$$y - 2z$$

Substitute the y expression for the y-variable.

$$\left(x^3 + 2x^2 + 1\right) - 2z$$

Substitute the z expression for the z-variable.

$$\left(x^3 + 2x^2 + 1\right) - 2\left(x^2 - 5x + 3\right)$$

Combine like terms.

$$\left(x^3 + 2x^2 + 1\right) + \ -2x^2 + 10x + \ -6$$

2 – Solve Unknown

$$x^3 + 10x - 5$$

Select answer choice D.

Ace Inequalitie 8

Multiply by a Negative

You should know that when you multiply or divide an inequality by a negative number, the sign of the inequality flips. For example, look at what happens when we multiply the following inequality by −2.

$$-3 < x + 3 < 3$$
$$-3(-2) < x + 3(-2) < 3(-2)$$
$$6 > -2x + \ -6 > -6$$

By convention, the SAT will write inequalities using the “less than” form. Therefore, instead of writing 6 is greater than the expression and the expression is greater than −6, write that −6 is less than the expression and that the expression is less than 6. For example:

$$-6 < -2x + \ -6 < 6$$

You can then solve for x:

$$0 < -2x < 12$$
$$0 > x > -6$$
$$-6 < x < 0$$

Manipulate Inequalities

Similar to manipulating equations, manipulating inequalities requires students to notice how two expressions or inequalities may be related to one another. For example: $-3 < x + 3 < 3$

Given the inequality above, what is one possible value of −8x − 24?

$$(-8)(-3) < (-8)(x+3) < (-8)(3)$$
$$24 > -8x + \ -24 > -24$$

Most students would not realize that the original inequality could have just been multiplied by −8 in order to get the expression we are looking for. High school math has trained us to be very formulaic with our though processes: find x first, then solve the rest. However, if you can notice relationships such as the one above, the SAT will reward you.

Convert Word Problems to Equations

You should be able to read a word problem and translate it into an inequality.

A banana has 98 calories. An apple has 77 calories. The daily recommended calories from fruits is 400 calories. Write an inequality that represents the possible number of bananas (b) and number of apples (a) to meet or exceed the recommended daily fruit calorie intake.

$$77a + 98b \geq 400$$

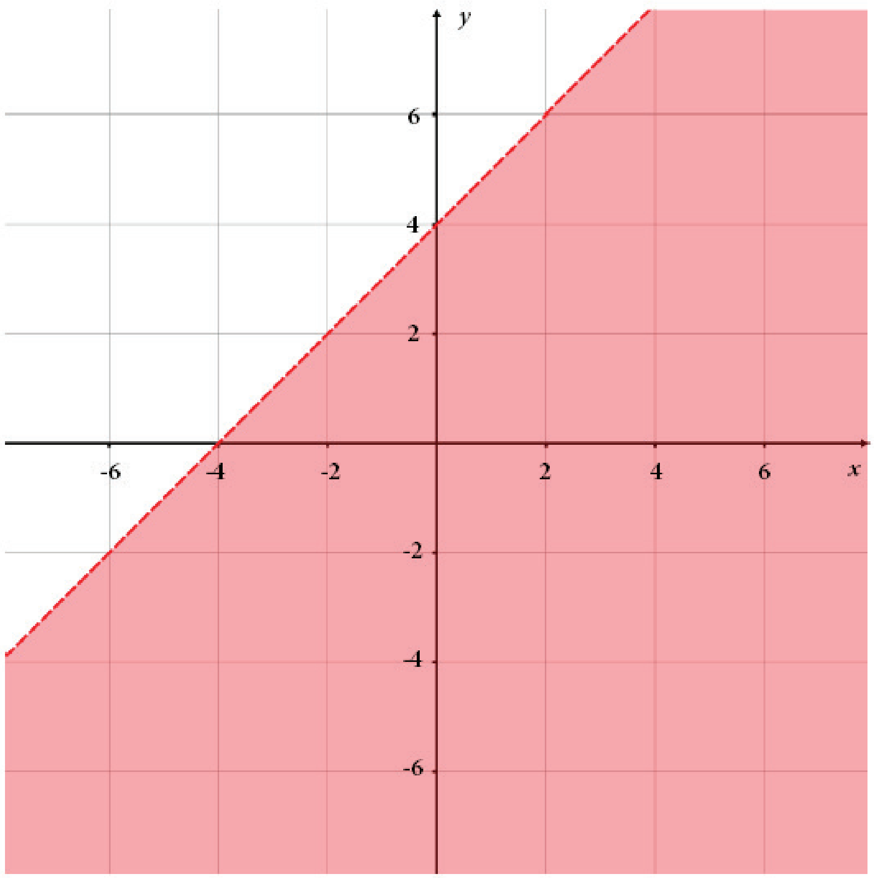
where: a = # of apples

b = # of bananas

Graphing Inequalities

You should be able to graph inequalities.

Graph $y < x + 4$



Any (x,y) value that falls in the red portion of the above graph is a possible solution to the inequality. For example, (2,2) is a solution but (0,6) is not a solution.

Expert Practice

1

John wants to buy at least 50 bottles of soda for a birthday party. Which of the following inequalities represents the possible number of six-packs, *s*, and four-packs, *f*, of soda John should buy?

(A) $6s + 4f > 50$

(B) $6s + 4f \geq 50$

(C) $\frac{6}{s} + \frac{4}{f} > 50$

(D) $\frac{6}{s} + \frac{4}{f} \geq 50$

Solution

1 – Ace Inequalities

When converting word problems to equations or inequalities, the first step should be to define your variables. Luckily, in this case, the problem has already defined the variables for you.

s = # of six – packs

f = # of four – packs

Next, decide what everything in the inequality will be in terms of. To help you decide, look at the given number that inequality has to be greater than or less than. In this case, that number is 50. Ask yourself, what does the 50 represent? Bottles!

Once you have determined that this inequality will be in bottles, ask yourself how many bottles should you multiply each variable by? There are six bottles in a six-pack, so multiply s by 6. There are four bottles in a four-pack, so multiply f by 4.

$$6s + 4f$$

Finally, determine if the expression needs to be greater than or less than the number given in the problem. In this case, the problem states "at least," which means you need 50 bottles or more; in other words, ≥ 50.

$$6s + 4f \geq 50$$

2 – Solve Unknown

Select answer choice B.

2

If $-\frac{3}{4} < 1 - 2t < -\frac{1}{4}$, what is one possible value of $8t - 4$?

Solution

1 – Ace Inequalities

This is a Free Response question, so we don't have the answer choices to help us. Instead of diving directly into the problem to solve for t as most students would, let's take a step back to think critically about the problem. Is there a way that I can manipulate the inequality to directly get the expression? Yes. Multiply the entire inequality by –4. So let's start by doing that.

$$-\frac{3}{4} < 1-2t < -\frac{1}{4}$$

$$(-4)\left(-\frac{3}{4}\right) < (-4)(1-2t) < (-4)\left(-\frac{1}{4}\right)$$

$$3 > -4+8t > 1$$

$$1 < -4+8t < 3$$

2 – Grid-In Answer

Fill in any number between 1 and 3 as your answer (the SAT will accept any answer that works for the inequality above). I would fill in 2.

Ace Exponents 9

Negative Exponents

You should be able to efficiently work with negative exponents.

$$9^{-2}$$

$$\frac{1}{9^2}$$

$$\frac{1}{81}$$

$$8^{-3}$$

$$\frac{1}{8^3}$$

$$\frac{1}{512}$$

Fractional Exponents

You should be able to efficiently work with fractional exponents.

$$9^{\frac{1}{2}}$$

$$\sqrt[2]{9}$$

$$3 \ \textit{or} \ -3$$

$$8^{-\frac{1}{3}}$$

$$\frac{1}{\sqrt[3]{8}}$$

$$\frac{1}{2}$$

Multiply by the Reciprocal

When you have a fractional exponent in an equation, you can get rid of the fraction by raising each side of the equation to the exponent's reciprocal.

$$8^{-\frac{1}{3}}$$

$$8^{-\frac{1}{3}\left(-\frac{3}{1}\right)}$$

$$8$$

Exponent to Exponent

You should know that when an exponent is raised to another exponent, you multiply.

$$\left(3^2\right)^3$$

$$3^6$$

$$729$$

Multiplying Common Bases

When you are multiplying common bases with exponents, add the exponents.

$$3^3 \times 3^2$$
$$3^5$$
$$243$$

Dividing Common Bases

When you are dividing common bases with exponents, subtract the exponents.

$$3^3 \div 3^2$$
$$3^1$$
$$3$$

Exponents with Variables

You should be able to apply all of the above principles with exponents to variables as well.

Exponent to Exponent with Variables

3^{x^y}

3^{xy}

x^{2^3}

x^6

Multiplying Common Bases with Variables

$3^x \times 3^y$

3^{x+y}

$x^3 \times x^2$

x^5

Dividing Common Bases with Variables

$3^x \div 3^y$ $x^3 \div x^2$

3^{x-y} x

Common Exponents

You should know some common exponents that appear often on the SAT.

Squares		
$1^2 = 1$	$6^2 = 36$	$11^2 = 121$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$

Cubes		
$1^3 = 1$	$3^3 = 27$	$5^3 = 125$
$2^3 = 8$	$4^3 = 64$	$6^3 = 216$

Exponents of 2		
$2^1 = 1$	$2^3 = 8$	$2^5 = 32$
$2^2 = 4$	$2^4 = 16$	$2^6 = 64$

Manipulate Exponents

You should be able to recognize numbers that are perfect squares or cubes.

$$\frac{27}{100}$$

$$\frac{3^3}{10^2}$$

Expert Practice

1

If $a^{\frac{1}{2}} = x^{-1}$, where $a > 0$ and $x > 0$, which of the following equations gives a in terms of x ?

(A) $a = \frac{1}{\sqrt{x}}$

(B) $a = \frac{1}{x^2}$

(C) $a = -\frac{1}{\sqrt{x}}$

(D) $a = -\frac{1}{x^2}$

Solution

1 – Ace Exponents

I will show you two ways to solve this problem. The first will be manipulation of exponents as we have just learned.

$$a^{\frac{1}{2}} = x^{-1}$$

First, multiply the exponent on a by it's reciprocal in order to get rid of the fractional exponent.

$$a^{\frac{1}{2}(2)} = x^{-1(2)}$$

$$a = x^{-2}$$

Next, apply your knowledge of negative exponents.

$$a = \frac{1}{x^2}$$

2 – Solve Unknown

Select answer choice B.

$$a^{\frac{1}{2}} = x^{-1}$$

Alternative Solution

1 – Substitute Abstract with Tangibles

For the variable a, strategically plug in a number that is easy to take the square root of.

$$4^{\frac{1}{2}} = x^{-1}$$

$$2 = \frac{1}{x}$$

$$x = \frac{1}{2}$$

Plug in $\frac{1}{2}$ into the answer choices.

(A) $a = \frac{1}{\sqrt{\frac{1}{2}}}$

(B) $a = \frac{1}{\left(\frac{1}{2}\right)^2}$

(C) $a = -\frac{1}{\sqrt{\frac{1}{2}}}$

(D) $a = -\frac{1}{\left(\frac{1}{2}\right)^2}$

2 – Select Answer

Select answer choice B. Notice that you don't have to solve all of the answer choices above. You should be able to quickly tell that only answer choice B would yield a = 4.

2

If $\sqrt[3]{a^{-\frac{1}{2}}} = x$, where $a > 0$ and $x > 0$, which of the following equations gives a in terms of x ?

(A) $a = \dfrac{1}{\sqrt[6]{x}}$

(B) $a = \dfrac{1}{x^6}$

(C) $a = \sqrt[6]{x}$

(D) $a = -x^6$

Solution

1 – Ace Exponents

I will show you two ways to solve this problem. The first will be manipulation of exponents as we have just learned.

$$\left(\sqrt[3]{a}\right)^{-\frac{1}{2}}$$

First, multiply the exponent on the cube root of a by it's opposite reciprocal in order to get rid of the fractional exponent.

$$\left(\sqrt[3]{a}\right)^{\frac{-1(-2)}{2}} = x^{-2}$$

Next, apply your knowledge of radicals.

$$\sqrt[3]{a} = x^{-2}$$

Multiply the exponent on a by its reciprocal in order to get rid of the fractional exponent.

$$a^{\frac{1}{3}} = x^{-2}$$

$$a^{\frac{1(3)}{3}} = x^{-2(3)}$$

Next, apply your knowledge of negative exponents.

$$a = x^{-6}$$

$$a = \frac{1}{x^6}$$

2 – Select Answer

Select answer choice B.

Alternative Solution

1 – Substitute Abstract with Tangibles

For the variable a, strategically plug in a number that after you take the cube root of it, you will then be able to easily take the square root of.

$$\left(\sqrt[3]{a}\right)^{-\frac{1}{2}} = x$$

$$\left(\sqrt[3]{64}\right)^{-\frac{1}{2}} = x$$

$$\left(4\right)^{-\frac{1}{2}} = x$$

$$x = \frac{1}{\sqrt{4}}$$

$$x = \frac{1}{2}$$

Plug in — into the answer choices.

(A) $a = \dfrac{1}{\sqrt[6]{\frac{1}{2}}}$

(B) $a = \dfrac{1}{\left(\frac{1}{2}\right)^6}$

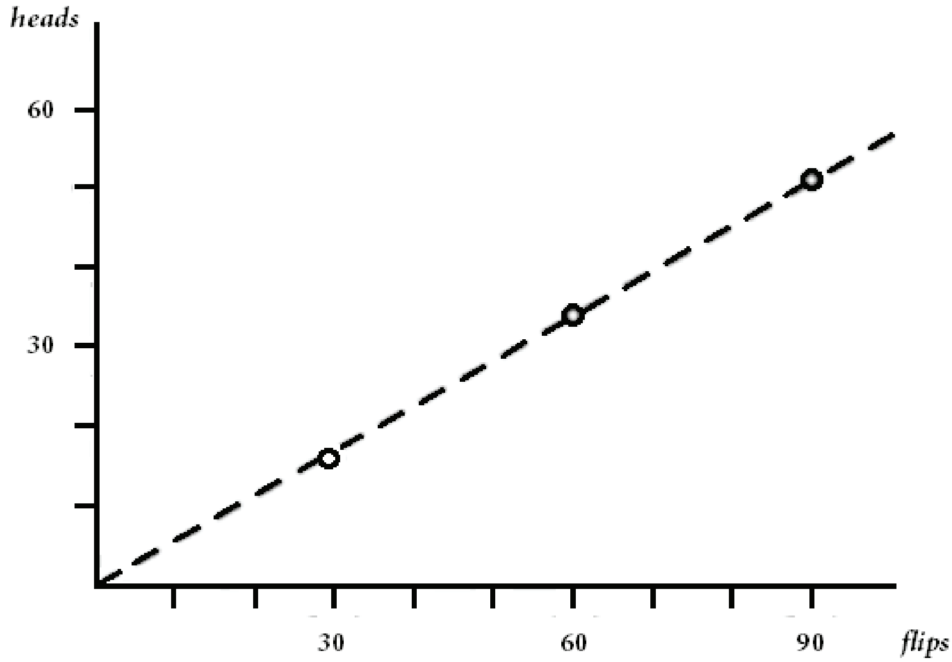
(C) $a = -\left(\dfrac{1}{2}\right)^6$

2 – Select Answer

Select answer choice B. Notice that you don't have to solve all of the answer choices above. You should be able to quickly tell that only answer choice B would yield a = 64.

Ace Data Analysis 10

Line of Best Fit



Whenever you're given a graph such as the one above, you should interpret one point. The first point says that when a coin was flipped 30 times that approximately 15 heads were obtained. Now let's try to answer a series of questions related to the above graphic.

(1) How many trials were there?

> **Each point on the graph represents a TRIAL of coin flips. Note that this is different than coin flips. There are only 3 points on the graph. Therefore, there were 3 trials.**

(2) How many coin flips were there?

Each point on the graph represents a TRIAL of coin flips. Each trial had a certain number of coin flips:

Trial 1 – 30 coin flips
Trial 2 – 60 coin flips
Trial 3 – 90 coin flips

Therefore, there were a total of 180 coin flips.

(3) How many trials differed by more than 5 heads from the heads predicted by the line of best fit?

All of the trials touch the line of the best fit. Therefore, no trial differed by more than 5 heads from the line of best fit.

(4) What does the line of best fit represent?

A line of best fit attempts to predict the change in y with respect to the change in x. Therefore, this line of best fit attempts to predict the number of heads that will occur after a certain number of coin flips.

(5) Based on the line of best fit, how many heads should you expect if you flip a coin 10 times?

You should find 10 on the x-axis. Then find the y-value that corresponds to that on the line of best fit. In this case, we would expect 5 heads.

(6) Based on the line of best fit, what is the average number of heads you would expect per coin flip?

Per 1 coin flip we would expect 0.5 heads. This is obtained by multiplying the $\frac{5}{10}$ from the last problem by the 1 coin flip in this problem.

Tables

College Major	Gender		Total
	Men	Women	
Humanities	4	10	14
Natural Sciences	11	10	21
Social Sciences	8	14	22
Total	23	34	57

Whenever you're given a table such as the one above, you should interpret one line. The first line says there are a total of 14 people with a humanities major: 4 men and 10 women. Now let's try to answer a series of questions related to the above graphic.

(1) What fraction of humanities and social sciences majors are female?

First I would count the total number of humanities and social science majors since the question wants a fraction of that: 36. Next, count the number of female humanities and social science majors: 24. Calculate the fraction.

$$\frac{24}{36} = \frac{2}{3}$$

(2) Which major has the highest percentage of females?

I would try to quickly ballpark this.

Humanities	**–**	**Huge Majority of Females to Males**
Natural Sciences	**–**	**About 50/50 of Females to Males**
Social Sciences	**–**	**Slight Majority of Females to Males**

Clearly, the humanities.

(3) Based on the national average that 50% of humanities majors go on to do graduate school, how many women can we expect to do graduate school from the humanities majors listed in the table (round to the nearest whole number)?

We would simply take 50% of the female humanities majors. 50% of the 10 female humanities majors is 5. We can expect 5 female humanities majors to go on to do graduate school.

Data Statements

A physician wanted to know if there is a correlation between playing video games and academic grades for the population of 9th - 12th graders in the United States. She collected survey responses from a random sample of 5000 9th - 12th graders in the United States that found convincing evidence that there is a negative correlation between video game use and academic grades.

Whenever you're given a statement such as the one above, we should quickly interpret what it means: The physician found that as video game use increased, academic grades decreased for 9th – 12th graders in the United States.

(1) Does increased video game use cause lower academic grades?

There is a difference between correlation and causation. Correlation implies association. Causation implies one item produces the other. Although increased video game use is correlated with lower academic grades, we don't know that it was the increased video game use that caused the lower academic grades.

(2) Does decreased video game use cause higher academic grades?

Once again, there is a difference between correlation and causation.

(3) Do higher academic grades cause lower use of video games?

Once again, there is a difference between correlation and causation.

(4) Do lower academic grades cause higher use of video games?

Once again, there is a difference between correlation and causation.

(5) Is there a positive association between video game use and academic grades?

Correlations are associations. Now that this question asks about associations, rather than causations, we know it is headed in the right direction. However, the association between

video game use and academic grades is not positive. This would imply that the more students use video games, the higher students' academic grades will be. Certainly, the data statement does not imply this correlation.

(6) Is there a negative association between video game use and academic grades for all 9th - 12th graders in the world?

> **Correlations are associations. Now that this question asks about associations, rather than causations, we know it is headed in the correct direction. In addition, the direction of the association is correct here: negative. This would imply that the more students use video games, the lower students' academic grades will be. The data statement does imply this. However, there is an incorrect qualification in the above question: "in the world." The data statement clearly states that the correlation it found applies only to 9th and 12th graders in the United States. Therefore, the data statement does not imply the qualification in this correlation.**

(7) Is there a negative association between video game use and academic grades for 9th - 12th graders in the United States?

→ **Yes! The data statement implies that there is a negative correlation between video game use and academic grades. This would imply that the more students use video games, the lower students' academic grades will be. In addition, the qualifier "in the United States" also agrees with the original data statement.**

Other Important Information Related to Data Statements

→ **The population in a survey/experiment must be well-defined**

Ex. A study on people with high blood sugar cannot be generalized to people with diabetes because the study did not define whether "high blood sugar" meant that the people had a fasting blood glucose level of 126mg/dL – the definition of diabetes.

→ **Participants in a survey/experiment must be randomly selected in order to generalize results to the greater population.**

Ex. If only people who visit the hospital are surveyed, the results cannot be generalized to the population of an entire town.

Participants in a survey/experiment must be randomly assigned to treatment groups in order to draw conclusions about cause and effect.

Ex. If people with a blood sugar above 126mg/dL are given a certain diabetes medication, but people with a blood sugar below 126mg/dL are not given that diabetes medication, a conclusion about the effectiveness of the treatment in the entire population cannot be drawn. Instead, researchers should randomly assign the treatment without any bias in order to make conclusions about cause and effect.

Expert Practice

The scatter plot below shows the relationship between the work experience of 8 employees and their weekly salaries. The line of best fit is also shown.

1

How many of the eight employees have an actual weekly salary that differs by more than $150 from the weekly salary predicted by the line of best fit?

(A) 2

(B) 3

(C) 4

(D) 6

Solution

1 – Ace Data Analysis

Let's start by interpreting one point on the above graph. The first point implies that worker with 1 year of work experience has a weekly salary of $550.

Next, let's think about what the line of best fit represents. Remember that the line of best fit predicts the change in y based on the change in x. In this case, the line of best fit would predict the increase in weekly salary based on the number of years of work experience of an employee.

Finally, let's find how many data points are more than $150 away from the line of best fit. This will tell us how many actual salaries are more than $150 away from the predicted salaries.

Employee With 2 Years of Work Experience – differs by approximately $160
Employee With 6 Years of Work Experience – differs by approximately $160
Employee With 10 Years of Work Experience – differs by approximately $210

2 – Select Answer

Select answer choice B.

2

Which of the following is the best interpretation of the slope of the line of best fit in the context of this problem?

(A) The predicted weekly salary in dollars of an employee with 0 years of work experience.

(B) The predicted work experience in years of an employee with $0 weekly salary.

(C) The predicted weekly salary increase in dollars for 1 year increase in work experience.

(D) The work experience increase in years for every dollar increase in weekly salary.

Solution

1 – Ace Data Analysis

We already solved this question while we were answering the last question:

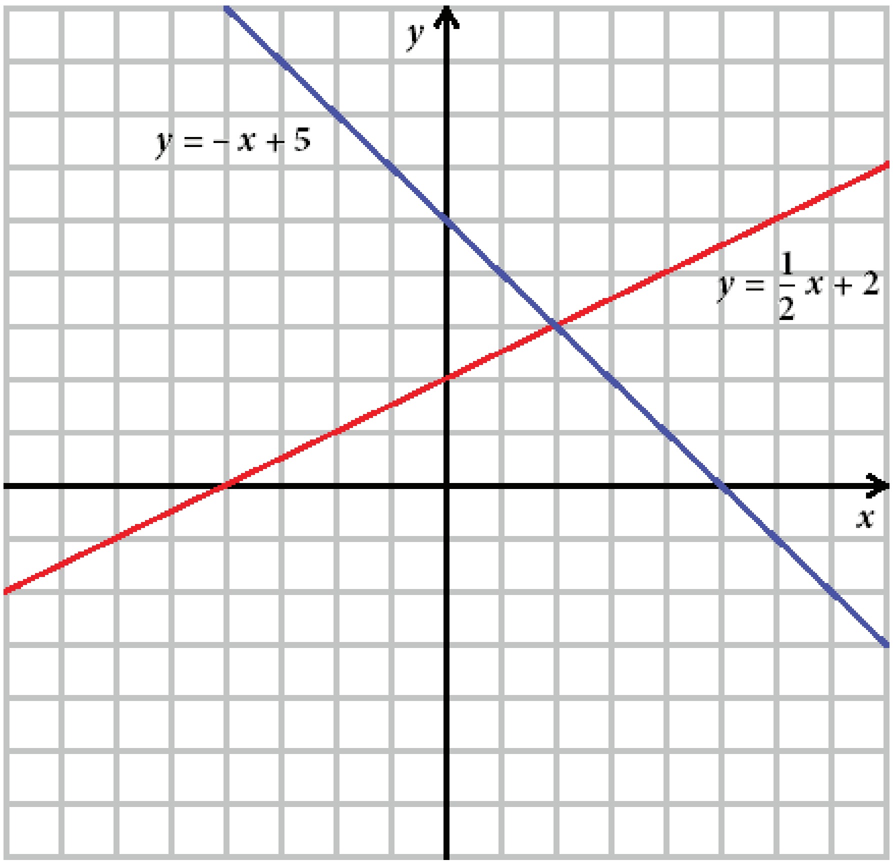
The line of best fit predicts the change in y based on the change in x. In this case, the line of best fit would predict the increase in weekly salary based on the number of years of work experience of an employee.

In particular, the line of best fit predicts the increase in weekly salary based on a 1-year change in the work experience (since the units of the x-axis are in increments of 1). In addition, we say “increase” here because the slope of the line of best fit is positive. If the slope of the line of best fit was negative, then we would say “decrease.”

2 – Select Answer

Select answer choice C.

Ace Graphs 11



Slope

Slope is defined as the rise over the run of a graph. You can find the slope of a graph by counting how many units a graph moves up/down over how many units a graph moves left/right. For example, the red line moves up 2 units for every 4 units that it moves to the right. Therefore, the slope of the red line is $\frac{2}{4}$ or $\frac{1}{2}$. The blue line moves down 3 units for every 3 units that it moves to the right. Therefore, the slope of the blue line is $-\frac{3}{3}$ or −1.

You can also find the slope of a linear graph anytime you have two points (x_1, y_1) and (x_2, y_2). Use the following formula to calculate slope:

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

For example, the red line passes through the points (–4,0) and (0,2). Therefore, the slope of the red line can be calculated by doing the following:

$$slope = \frac{2-0}{0 - -4}$$

$$slope = \frac{2}{4}$$

$$slope = \frac{1}{2}$$

The blue line passes through the points (5,0) and (0,5). Therefore, the slope of the blue line can be calculated by doing the following:

$$slope = \frac{5-0}{0-5}$$

$$slope = \frac{5}{-5}$$

$$slope = -1$$

Finally, you can calculate slopes when lines are in the slope-intercept form:

$$y = mx + b$$

Whenever you are working with linear graphs on the SAT, you should attempt to put equations in the slope-intercept form. This will not only give you a tremendous amount of information about the graph, but also often help you solve the problem.

The red line has the following, slope-intercept form:

$$y = \frac{1}{2}x + 2$$

$$m = slope = \frac{1}{2}$$

The blue line has the following, slope-intercept form:

$$y = -x + 5$$

$$m = slope = -1$$

*** It's very important to note that any line that passes through the origin has a y-intercept of 0. In other words, b = 0. Although this piece of information may seem trivial, you will find it exceptionally useful on the SAT.**

Parallel Slope

Parallel lines have equal slopes.

→ **A line that is parallel to the red line would have a slope of** $\frac{1}{2}$

→ **A line that is parallel to the blue line would have a slope of –1**

Perpendicular Slope

Perpendicular lines have slopes that are opposite reciprocals of one another. I think of "opposite reciprocal" as flipping the fraction and changing the sign.

→ **A line that is perpendicular to the red line would have a slope of –2**

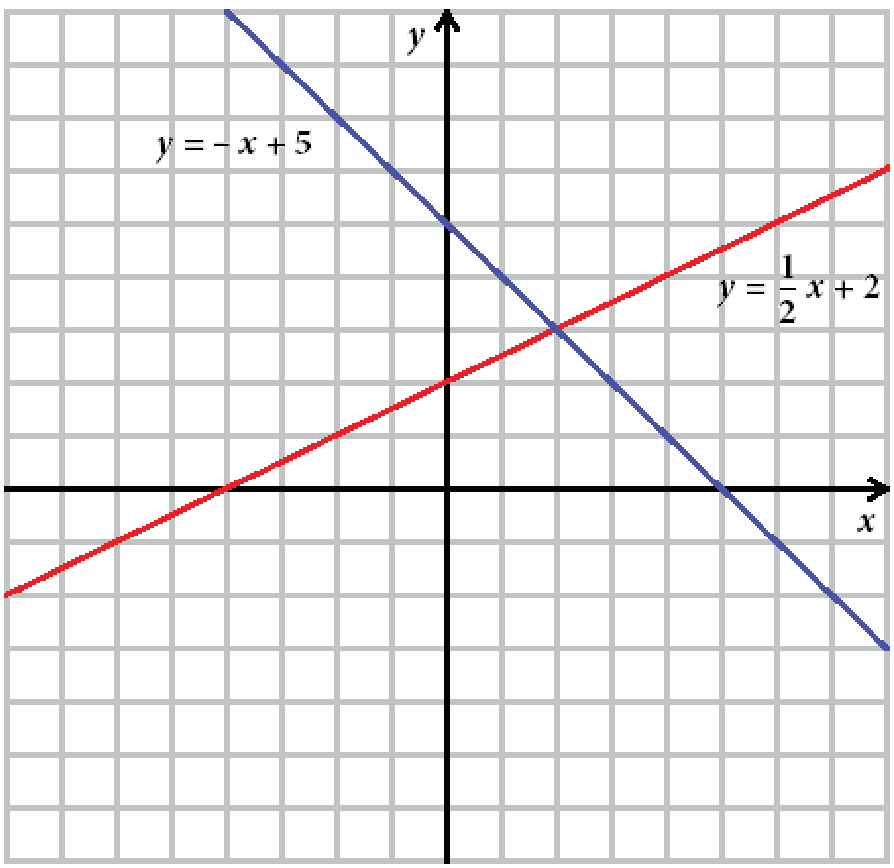
→ **A line that is perpendicular to the blue line would have a slope of +1**

Shifts

The slope of a graph does not change with shifts.

Slope Magnitude

The magnitude of a slope is equivalent to its steepness. For example, a slope of 2 has a greater magnitude than a slope of $\frac{1}{2}$. Similarly, a slope of –2 has a greater magnitude than a slope of $-\frac{1}{2}$



Intercepts

The x-intercept of a graph indicates the x-value when the graph crosses the x-axis (essentially when the $y = 0$ since the graph is not moving up or down vertically).

The y-intercept of a graph indicates the y-value when the graph crosses the y-axis (essentially when $x = 0$ since the graph is not moving left or right horizontally).

Domain & Range

Domain – All possible x-values of a graph.

Range – All possible y-values of a graph.

Maximum & Minimum

Maximum – The greatest y-value of a graph.

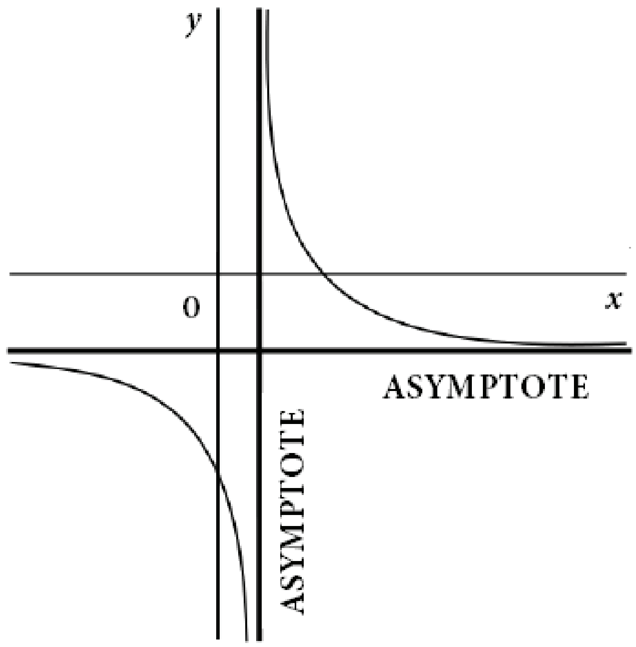
→ If dealing with a parabola that opens downward, the maximum value is the vertex.

Minimum – The lowest y-value of a graph.

→ If dealing with a parabola that opens upward, the minimum value is the vertex.

Asymptotes

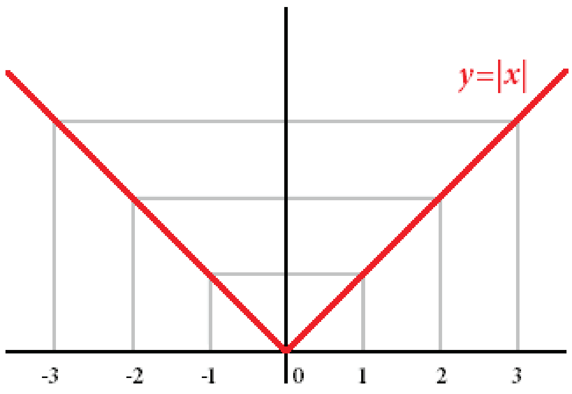
A line that a curve approaches, but does not touch. For example, the diagram below has a horizontal asymptote at 0 and a vertical asymptote at approximately 1.



Even & Odd Functions

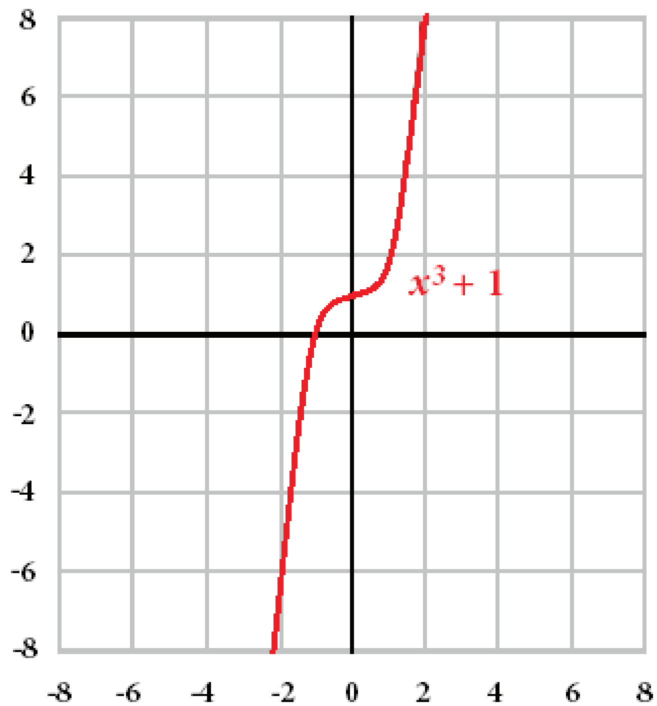
Even Functions – Symmetrical about the y-axis → f(–x) = f(x)

→ For example: Even functions are |x|, x^2, x^4, and cos(x).



Odd Functions – Symmetrical about the x-axis→ –f(x) = f(–x)

→ **For example: Even functions are x, x^3, and sin(x).**



Up/Down Shifts

If a number is added to outside of the x, the graph will move up that number of units.

→ **For example, let's add 4 to the red line:**

$$y = \left(\frac{1}{2}x + 2\right) + 4$$

This would shift the red line vertically up 4 units on the graph.

If a number is subtracted from the outside of x, the graph will down that number of units.

→ **For example, let's subtract 4 from the blue line.**

$$y = (-x + 5) - 4$$

This would shift the blue line vertically down 4 units on the graph.

Left/Right Shifts

If a number is added to inside of the x, the graph will move left that number of units.

→ **For example, let's add 4 inside of the x to the red line:**

$$y = \frac{1}{2}(x + 4) + 2$$

This would shift the red line horizontally to the left 4 units on the graph.

If a number is subtracted from the inside of x, the graph will move right that number of units.

→ **For example, let's subtract 4 inside of the x from the blue line.**

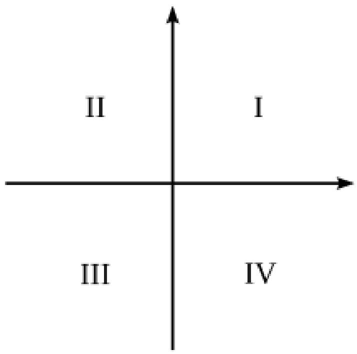
$$y = (-x - 4) + 5$$

This would shift the blue line horizontally to the right 4 units on the graph.

Reflections

When a graph is reflected about the x-axis, you simply flip it across the x-axis. When a graph is reflected about the y-axis, you simply flip it across the y-axis.

Quadrant Numbers

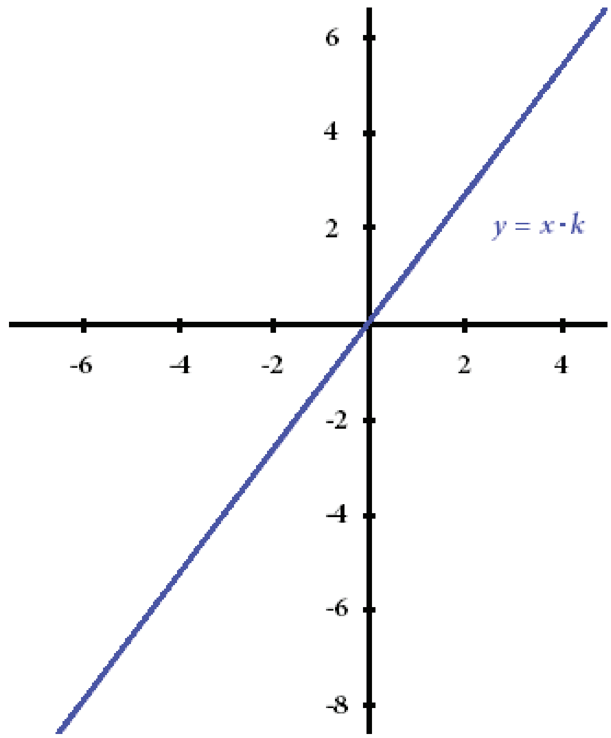


y = x

Dogs	Cats
5	5
3	5
9	9
0	3

Suppose that dogs were plotted on the x-axis and cats on the on the y-axis. How many data points would be above the line y = x ?

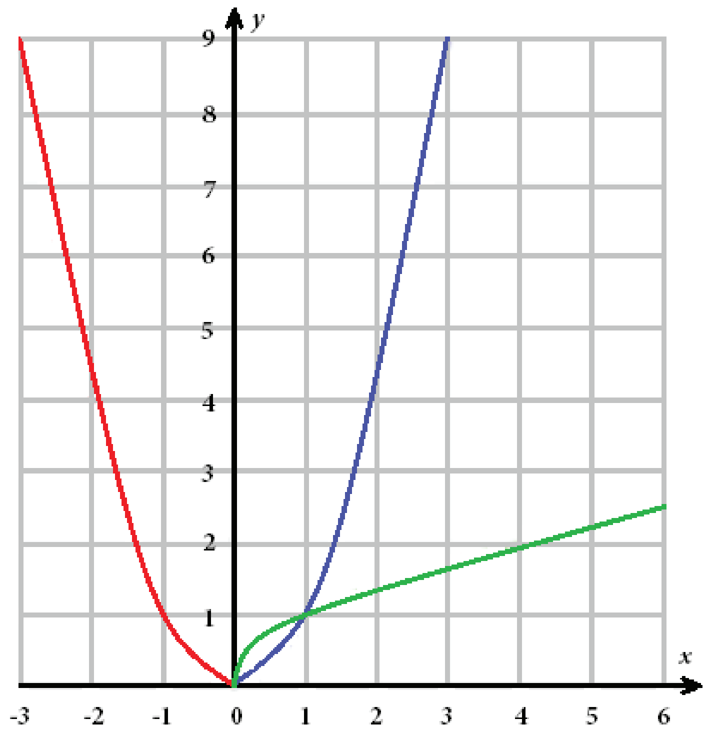
You should be familiar with how the graph of y = x looks.



There would be two data points for which the y-value is greater than the x-value (in other words, above the y = x line): (3, 5) and (0, 3)

Graph Types

Squares/Roots



Standard Form of Quadratic Equations

$$y = ax^2 + bx + c$$

If a is +, the parabola opens upwards

If a is –, the parabola opens downwards

The equation for the quadratic pictured above would be the following:

$$y = x^2$$

Notice how the coefficient a on x^2 is a positive number: 1. This is why the graph opens upwards.

Vertex Form of Quadratic Equations

If a is +, the parabola opens upwards

If a is –, the parabola opens downwards

(h, k) represents the vertex of the parabola

The equation for the parabola pictured above would be the following:

$$y = x^2$$

Notice how the coefficient a on x^2 is a positive number: 1. This is why the graph opens upwards.

In addition, there is no number subtracted from the inside of x, nor is there a number added outside of x. Therefore, the vertex is at (0, 0).

How to Find the Vertex Given The Zeros

If you know the zeros of a quadratic equation, you should be able to find the vertex.

$$y = 2(x - 4)(x + 6)$$

The zeros of the above equation are x = 4 and x = –6. But the actual coordinates of these zeros are (4, 0) and (–6, 0). The x-coordinate of the vertex of the parabola will be halfway in between these two zeros since a parabola is symmetrical.

$$x = \frac{4 + -6}{2} = -1$$

To find the y-coordinate of the vertex, simply plug in x = –1 into the original equation.

$$y = 2(-1-4)(-1+6)$$
$$y = 2(-5)(5)$$
$$y = -50$$

The vertex of the parabola is at (–1, -50).

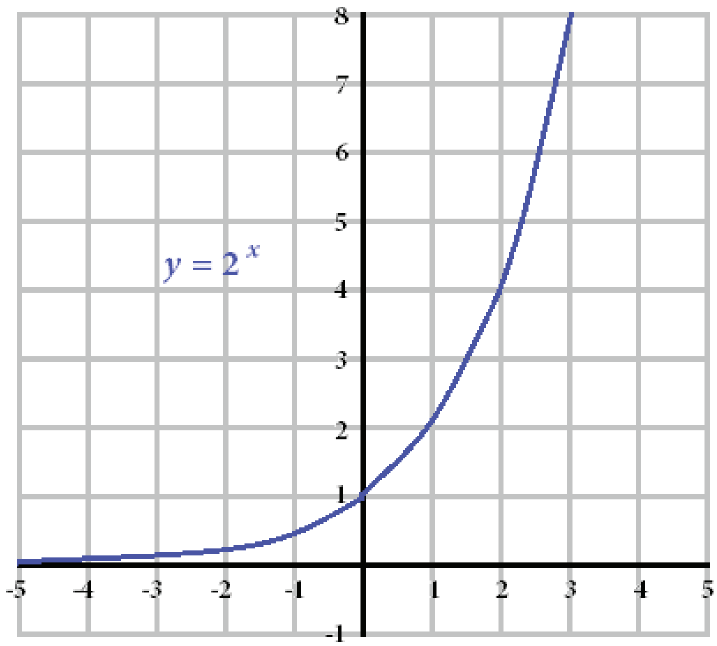
Root Equations

You should also be familiar with how square root graphs look. The green line in the above image is a good example. It is the graph of the following equation:

$$y = \sqrt{x}$$

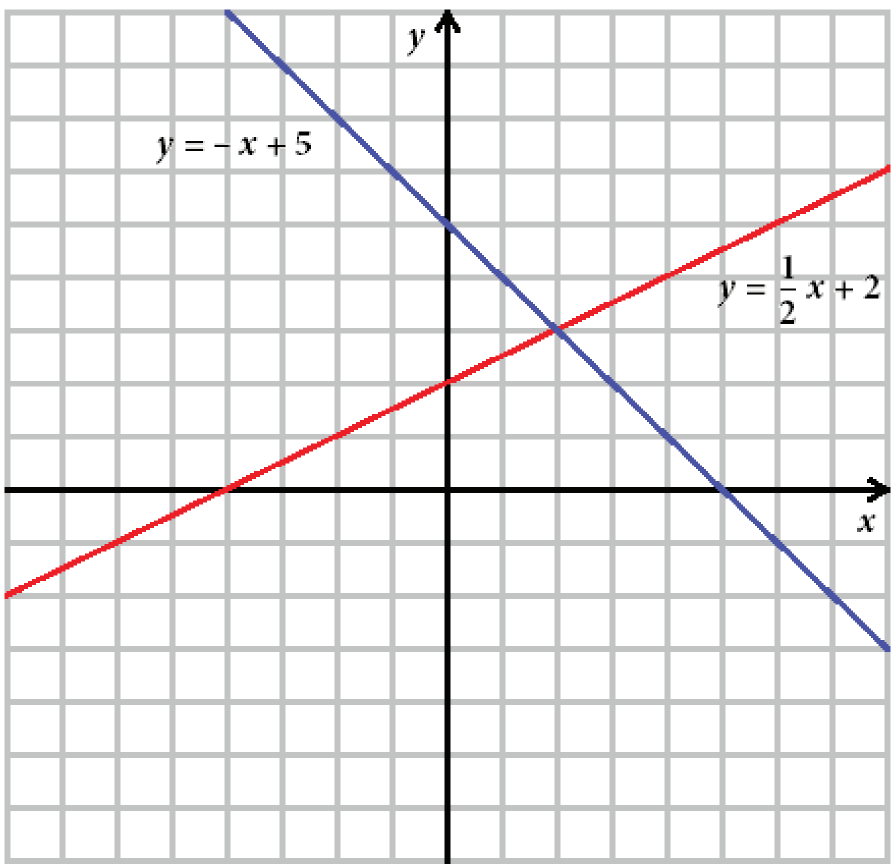
Exponential Growth Graphs

You should be familiar with graphs that show exponential growth. Below is a good example.



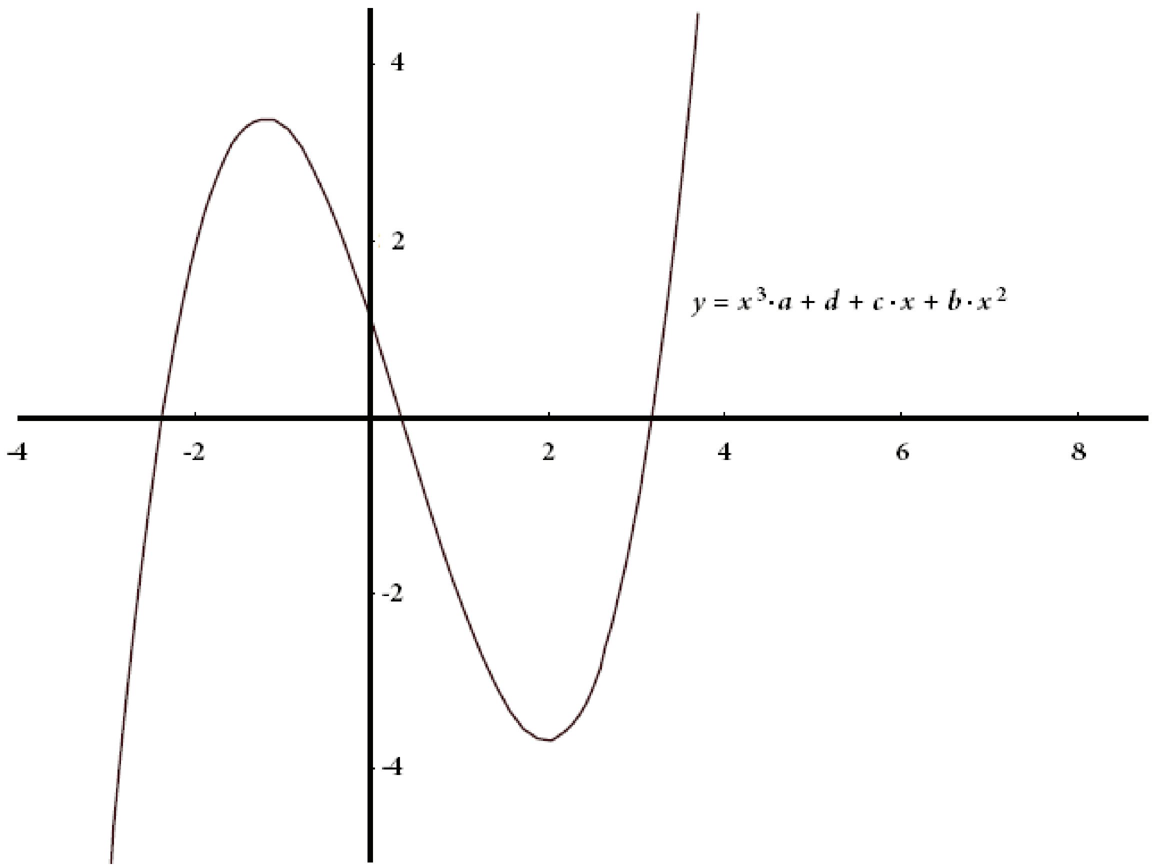
Intersections

When two graphs intersect, that is a "solution" for that system of equations. For example, a solution to the system of equations shown below is approximately (2, 3).

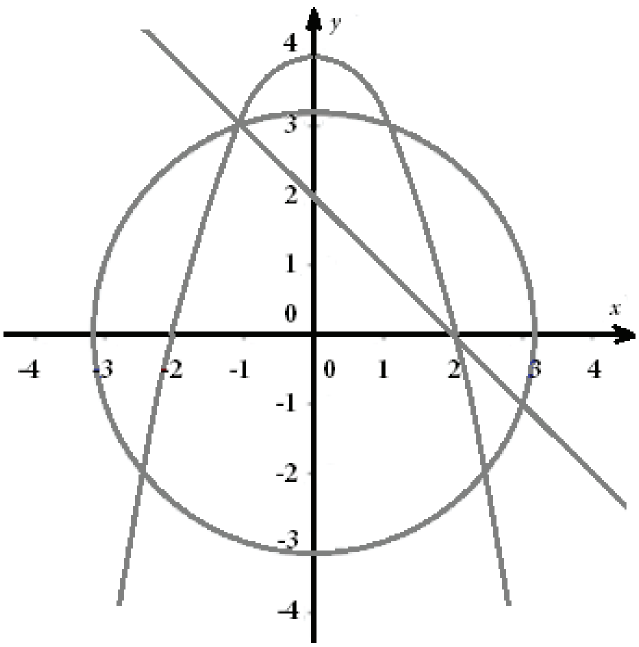


Zeros

The "zeros" of a function represent the x-values at which a graph's y-value is 0. Visually, "zeros" are when a graph crosses the x-axis. The below graph has 3 distinct zeros (approximately –2.2, 0.2, and 3).



Expert Practice



1

A system of three equations and their graphs in the *xy*-plane are shown in the picture above. How many solutions does the system have?

$$\begin{cases} x^2 + y^2 = 10 \\ y = 4 - x^2 \\ y = 2 - x \end{cases}$$

(A) One

(B) Two

(C) Three

(D) Four

Solution

1 – Ace Graphs

Although this problem appears complicated, it really isn't! We just covered that the solution to a system of equations is anywhere the graphs intersect. Well, there is only one point that all three graphs intersect: (–1, 3). Therefore the answer is 1.

2 – Select Answer

Select answer choice A.

2

What is the vertex of parabola $y = (2x + 6)(x + 1)$?

(A) (3, 1)

(B) (–3, –1)

(C) (–2, –2)

(D) (2, –2)

Solution

1 – Ace Graphs

In order to find the vertex of a parabola, we need to put the quadratic equation in vertex form.Let's start by multiplying the expressions in the equation together.

$$y = (2x + 6)(x + 1)$$
$$y = 2x^2 + 2x + 6x + 6$$
$$y = 2x^2 + 8x + 6$$
$$y = 2(x^2 + 4x + 3)$$

This is the most complicated part of changing the quadratic equation to vertex form. You must imagine that the equation does not have the constant at the end. In this case, you might think that constant is 3. However, 3 is being multiplied by 2, so the constant is actually 6. Let's start by pulling 6 out of the equation. Then, we need to get rid of the 6 from the right side of the equation. In order to do this, subtract 6 from both sides of the equation.

$$\begin{aligned} y &= 2(x^2 + 4x) + 6 \\ -6 & \qquad\qquad\quad -6 \\ y - 6 &= 2(x^2 + 4x) \end{aligned}$$

Now we need to complete the square. This means creating a perfect square out of the $x^2 + 4x$ that we have. The best way to do this is to take half of the middle coefficient (in this case, half of 4 is 2), square it (in this case, 2 squared is 4), and add it to the end of the $x^2 + 4x$.

$$y - 6 = 2(x^2 + 4x + 4)$$

However, am I really adding 4? No! I am actually adding 8 since it is multiplied by 2. In addition, if I add 8 to the right side of the equation, I must add 8 to the left side of the equation to keep everything even.

$$y - 6 + 8 = 2(x^2 + 4x + 4)$$

Now write the quadratic expression on the right side of the equation as a perfect square.

$$y + 2 = 2(x + 2)^2$$

$$y = 2(x + 2)^2 + -2$$

We finally have the quadratic equation in vertex form: $y = a(x - h)^2 + k$

Therefore, the vertex of this parabola is: $(-2, -2)$

2 – Select Answer

Select answer choice C.

Ace Center of Data 12

Mean

To find the mean (or average) of a set of numbers, add all of the numbers together and divide by the number of items in the set.

→ **Find the mean of the following set of numbers { 1, 2, 3, 4, 4, 5 }**

$$\frac{1+2+3+4+4+5}{6}$$

$$\frac{19}{6}$$

$$3.17$$

Median

To find the median (or middle) of a set of numbers, arrange all of the numbers from smallest to largest and select the middle value. If there is an even number of items in the set, take the average of the two middle values.

→ **Find the median of the following set of numbers { 1, 2, 3, 4, 4, 5 }**

$$\frac{3+4}{2}$$

$$3.5$$

→ Find the median number of dogs owned by all students at both Clark High School & Coronado High

School.

Number of Dogs Owned	Clark High School	Coronado High School
0	400	390
1	320	380
2	420	280
3	431	380

Total # of Students → 3,001

The Median Will Be → The number of dogs owned by the 1501st student

# of students with 0 dogs? → 790

# of students with 1 dog? → 700

# of students with 2 dogs? → 700

Therefore, the median number of dogs is 2. This is because the 1501st student will certainly be a student with 2 dogs (students with 2 dogs make up students 1491 – 2190 when the students are arranged from smallest to largest number of dogs owned)

Mode

To find the mode of a set of numbers, select the number that appears most often.

→ Find the mode of the following set of numbers { 1, 2, 3, 4, 4, 5 }

4

Range

To find the range of a set of numbers, subtract the smallest number from the largest number.

→ Find the range of the following set of numbers { 1, 2, 3, 4, 4, 5 }

5 – 1

4

Standard Deviation

Standard deviation is a measure of how far the values in a data set are from the mean of that data set. You will not be required to calculate standard deviation on the SAT, but you do need to be able to compare two data sets to see which will have a higher standard deviation.

→ Which of the following two sets of data has a larger standard deviation?

Set A: { 1, 2, 3, 4, 4, 5 }

Set B: { 1, 2, 3, 99, 98, 100 }

Set B has a larger standard deviation because the values in the data set are spread out far from the mean (50.5) whereas the values in Set B are near the mean (3.2).

Sum of Means

Sum of Means = Average x # of Items in Set

→ What is the last value in a set of 10 numbers that has an average of 35 and the other 9 numbers

are 1, 2, 3, 4, 5, 6, 7, 8, 9?

Sum Of Means = 35 x 10

Sum of Means = 350

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45

Last Value → 350 – 45 = 305

Outliers

In a symmetrical distribution (ex. normal distribution) of data, the mean = median. However, if there are outliers – values that are significantly smaller or larger than the rest of the data – the mean will be pulled in the direction of the outliers. Outliers do not affect the median.

If the mean car value is $50,000 and the median car value is $30,000 out of a set of 5 cars, there is likely an outlier car (or cars) that is more expensive than most of the cars in the set.

Example: $20,000, $20,000, $30,000, $35,000, $145,000

If the mean car value is $30,000 and the median car value is $50,000 out of a set of 5 cars, there is likely an outlier car (or cars) that is less expensive than most of the cars in the set.

Example: $0, $0, $50,000, $50,000, $50,000

Margin of Error

Margin of error describes how far off the sample mean is from the true mean of a population.

A larger standard deviation increases the margin of error of the average.

➝ Which of the following two sets of data has a larger margin of error?

Set A: { 1, 2, 3, 4, 4, 5 }

Set B: { 1, 2, 3, 99, 98, 100 }

Set B because it has the larger standard deviation.

A larger sample size reduces the margin of error of the average.

➝ A survey of 100 high school students found an average of 7 hours of sleep with a margin of error of 1 hour. Would a sample of 500 high school students reduce the margin of error?

Yes.

Confidence Interval

A 95% confidence interval represents the range of values for which you can be 95% confident that the true mean of a population lies in between them.

➝ What statement can you make if a researcher finds that the average lifespan of 1000 randomly

selected fire ants has a confidence interval between 4 and 6 weeks?

You can be 95% certain that the true average lifespan of all fire ants is between 4 and 6 weeks.

Adding Means

To find the mean of two sets of data that each of their own respective means, add all of the numbers together and divide by total number of units in the set.

The mean number of hours of sleep for 10 boys is 8 hours. The mean number of hours for 20 girls is 7 hours. What will the mean of the total group be?

$$10 \times 8 = 80 \text{ hours slept by boys}$$

$$20 \times 7 = 140 \text{ hours slept by girls}$$

$$80 + 140 = 220 \text{ hours slept total}$$

$$\frac{220 \text{ hours slept total}}{30 \text{ people}}$$

$$7.33 \text{ average hours slept}$$

Expert Practice

1

A farmer randomly selected 40 ducks on the farm and measured their weight. The mean weight in the sample was 24 pounds, and the margin of error was 2.8 pounds. The farmer intends to replicate the survey and will attempt to get a smaller margin of error. Which of the following samples will most likely result in a smaller margin of error for the estimated mean weight of ducks?

(A) 25 randomly selected ducks on the farm

(B) 25 randomly selected animals on the farm

(C) 100 randomly selected ducks on the farm

(D) 100 randomly selected animals on the farm

Solution

1 – Ace Center of Data

In order to reduce the margin of error, we would want to increase the sample size of ducks
(not animals).

2 – Select Answer

Select answer choice C.

2

In the class, the mean height of boys is 173 cm, and the mean height of girls is 165 cm. Which of the following must be true about the mean height, m, of all students in the class?

(A) $m < 169$

(B) $m = 169$

(C) $m > 169$

(D) $165 < m < 173$

Solution

1 – Ace Center of Data

When adding means, the new mean must be in between the two original means.

2 – Select Answer

Select answer choice D.

Ace Unit Conversions & Rates 13

Time Conversions

You must be able to convert from one unit of a time to another.

→ **How many seconds are in 1 day?**

$$1 \text{ day} = 24 \text{ hours}$$

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

$$1 \text{ day} = 24 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}}$$

$$1 \text{ day} = 24 \times 60 \times 60 \text{ seconds}$$

$$1 \text{ day} = 86{,}400 \text{ seconds}$$

Length Conversions

You must be able to convert from one unit of length to another.

→ **How inches are in 1 yard?**

$$1 \text{ yard} = 3 \text{ feet}$$

$$1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ yard} = 3 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}}$$

$$1 \text{ yard} = 36 \text{ inches}$$

Rates

You must be able to efficiently use rates.

→ **How many yards will a centipede travel in 1 day if it moves at a rate of 1 inch per 10 seconds?**

$$1\text{ day} = 86{,}400\text{ seconds}$$

$$\frac{86{,}400\text{ seconds}}{10\text{ seconds}} = 8{,}640\text{ inches}$$

$$8{,}640\text{ inches}$$

$$8{,}640\text{ inches} \times \frac{1\text{ yard}}{36\text{ inches}} = 240\text{ yards}$$

Expert Practice

1

John rides a bike at the average speed of 4 meters per second. He rides a maximum 12 hours every day. How many days does he need to pass 920 miles if he rides the max time everyday? (Note: 1 mile = 1.6 kilometers)

(A) 6

(B) 7

(C) 8

(D) 9

Solution

1 – Ace Unit Conversions

Typically, I like to convert bigger units into smaller units. So let's start by figuring out how many seconds are in 12 hours.

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

$$1 \text{ hour} = 60 \text{ minutes} \times \frac{60 \text{ seconds}}{1 \text{ minute}}$$

$$1 \text{ hour} = 3{,}600 \text{ seconds}$$

$$12 \text{ hours} = 43{,}200 \text{ seconds}$$

Now that we know that John rides a bike for 43,200 seconds per day, let's figure out how many meters he goes given that he rides at an average speed of 4 meters per second.

$$43{,}200 \text{ seconds} \times \frac{4 \text{ meters}}{1 \text{ seconds}}$$

$$172{,}800 \text{ meters}$$

Now that we know that John travels 172,800 meters per day, let's figure out how many kilometers that is given that 1,000 meters = 1 kilometer.

$$172{,}800 \text{ meters} \times \frac{1 \text{ kilometer}}{1{,}000 \text{ meters}}$$

$$172.8 \text{ kilometers}$$

Now that we know that John travels 172.8 kilometers per day, let's figure out how many miles that is given that 1 mile = 1.6 kilometers.

$$172.8\ \text{km} \times \frac{1\ \text{mile}}{1.6\ \text{km}}$$

$$108\ \text{miles}$$

Now that we know that John travels 108 miles per day, let's figure out how many days he would need to travel 920 miles.

$$920\ \text{miles} \times \frac{1\ \text{day}}{108\ \text{miles}}$$

$$8.5\ \text{days}$$

John would require 8.5 days to travel 920 miles if he rides a maximum of 12 hours per day at an average speed of 4 meters per second.

2 – Select Answer

Select answer choice D. The answer to this question is not 8. On the 8th day, John would have only travelled 864 miles (8 days x 108 miles per day). Therefore, John would need at least 9 days to travel 920 miles.

2

Sally is about to leave to work in 15 minutes. But first she needs to download 10 files of 62 Megabytes each onto her memory stick. Her download internet connection is 3.5 Mbps (Megabits per second). If 1 Megabit equals 0.125 Megabytes, what is the maximum number of files that Sally can fully download within the 15 minutes period?

(A) 5

(B) 6

(C) 10

(D) 0

Solution

1 – Ace Unit Conversions

Typically, I like to convert bigger units into smaller units. So let's start by figuring out how many seconds are in 15 minutes.

$$1 \text{ minute} = 60 \text{ seconds}$$

$$15 \text{ minutes} = 900 \text{ seconds}$$

Now that we know Sally has 900 seconds to download items, let's calculate how many megabits of data she can download given that her download speed is 3.5 megabits per second.

$$900 \text{ seconds} \times \frac{3.5 \text{ megabits}}{1 \text{ second}}$$

$$3{,}150 \text{ megabits}$$

Now that we know Sally can download 3,150 megabits of data, let's calculate how many megabytes of data that is given that 0.125 megabytes = 1 megabit.

$$3{,}150 \text{ megabits} \times \frac{0.125 \text{ megabyte}}{1 \text{ megabit}}$$

$$393.75 \text{ megabytes}$$

Given that Sally can download 393.75 megabytes of data, and each file is 62 megabytes big, let's calculate how many files she can download.

$$393.75 \text{ megabytes} \times \frac{1 \text{ file}}{62 \text{ megabytes}}$$

$$6.35 \text{ files}$$

Sally has time to download 6 full files in the 15-minute time period that she has.

2 – Select Answer

Select answer choice B.

Ace Ratios & Percentag 14

Percent Change

To calculate the percent change, subtract the new value from the original value, then divide by the original value.

→ **If there are 15 computers in stock on November 1st and 12 computers in stock on November 3rd,**

by what percent did the stock decrease?

$$\frac{15-12}{15}=.20$$

20%

Included Percentages

When a percentage is included in the total price, make x the price without the percentage. Multiply x by 1. _ _ where the blanks represent the percentage amount.

→

If $10 represents the price that includes a 8% fee when you make a purchase at Computer City,

what is the price without the fee?

$$1.08x = \$10$$

$$x = \$9.26$$

Adding Densities

City A has a population density of 75 people per square kilometer and an area of 10 square kilometers. City B has a population density of 100 people per square kilometer and an area of 22 square kilometers. What is the population density of City A and City B?

$$\text{City A: } \frac{75 \text{ people}}{\text{square km}} \times 12 \text{ square km} = 900 \text{ people}$$

$$\text{City B: } \frac{100 \text{ people}}{\text{square km}} \times 21 \text{ square km} = 2{,}100 \text{ people}$$

$$\text{City A + City B: } \frac{3{,}000 \text{ people}}{33 \text{ square km}} = 90.9 \text{ people per square km}$$

Linear vs. Exponential Growth

If the difference between quantities is constant over time, then you have a linear relationship. If the ratio between quantities is constant over time, then you have an exponential relationship.

Linear Growth - the difference between each quantity is a constant addition of 10

Time (Years)	Population
0	10
1	20
2	30
3	40

Exponential Growth - the ratio between each quantity is a constant multiplication of 10

Time (Years)	Population
0	10
1	100
2	1,000
3	10,000

Exponential Growth vs. Exponential Decay

Exponential growth represents an increase at a constant ratio over time.

$$y = a(1+r)^t$$

where y = new amount

a = initial amount

r = growth rate (as a decimal)

t = number of time intervals

Exponential decay represents a decrease at a constant ratio over time.

$$y = a(1-r)^t$$

where y = new amount

a = initial amount

r = growth rate (as a decimal)

t = number of time intervals

*** It's not necessary to memorize the above formulas because you can almost always Substitute Abstract with Tangibles to find the answer quickly. For example, think about how much of a substance is left after 2 years, then find an answer choice that matches.**

Simple vs. Compound Interest

Simple interest is only paid on the original amount of money deposited (also known as the "principal").

$$A = P(1 + rt)$$

where A = amount at the end of t years
P = original amount of money deposited (or "principal")
r = interest rate (written as a decimal)
t = time in years

→ How much money is in your bank account if you deposited $100 five years ago at an interest rate of 3%?

$$A = 100(1 + .03(5))$$

$$A = 100(1.15)$$

$$A = \$115$$

Compound interest is not only paid on the principal, but also on the previous interest earned.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

where A = amount at the end of t years
P = original amount of money deposited (or "principal")
r = interest rate (written as a decimal)
t = time in years
m = # of compounding periods per year

→ **How much money is in your bank account if you deposited $100 five years ago at an interest rate of 3% compounded semiannually?**

*** Note semiannually means that the interest is compounded twice a year, so m = 2.**

$$A = 100\left(1 + \frac{.03}{2}\right)^{2(5)}$$

$$A = 100(1.015)^{10}$$

$$A = \$116.05$$

Unknown Proportions

The SAT will sometimes ask you to add or subtract from a set in order to get some final ratio. In this case, you should make the addition/subtraction x, and solve for x.

→ **There are 12 dogs and 8 cats. If 5 more cats are added to the group, how many more dogs must be added so that 4/5 of the total number of animals in the study are dogs?**

$$\frac{\text{dogs}}{\text{total animals}} = \frac{2}{5}$$

$$\frac{12 + x}{20 + x} = \frac{4}{5}$$

$$5(12 + x) = 4(20 + x)$$

$$60 + 5x = 80 + 4x$$

$$x = 20$$

Working Backwards

You should be able to use information you already know to solve problems that involve percentages and proportions.

→ When you purchase a gift card of $1000 or less from Computer City, an 8% service fee is added to its $1.08x = \$1{,}000$ **price. If you purchase a $1000 gift card from Computer City, no fee is added, but you lose the money that is left on the gift card within 1 month. What is the least amount you would have to spend on the gift card within 1 month to be cheaper than simply paying the fee?**

$$x = \$925.93$$

This means that you should spend at least $925.93 for the gift card to be worth it. If you only spend $900, then you would have been better off simply paying the 8% fee (a total of $972) because you would have spent less than the $1000 you spent on the gift card. However, if you spend $950, then you are better off buying the gift card because paying the 8% fee would be more expensive ($1008) than the $1000 you paid for the gift card.

Data Interpretation

You should be able to interpret proportions and percentages given a set of data. .

→ A sample of 100 people found that 70 people gave a video game a 5-star review and 30 people gave the game a 3-star review. How many people gave the game a 5-star review based on the data below -

Left a Review	**-**	**13,138**
Did Not Leave a Review	**-**	**74,245**

Solution: The sample represents only people that left a review (5-star or 3-star). Therefore, we can multiply 70% by 13,138 to get 9,197.6 people who left a 5-star review. However, there's no such thing as 0.6 of a person. Therefore, 9,198 people left a 5-star review.

Expert Practice

1

Mia needed exactly 2,950 yen to buy a shirt in Tokyo, so she went to the bank to change dollars into yen. The bank charges a 4% fee on every currency exchange transaction, and Mia gave the bank \$26. If the number of yen Mia received was exactly the number she needed to buy the shirt, what foreign exchange rate, in yen per one U.S. dollar, did the bank use for Mia's transaction?

Solution

1 – Ace Ratios & Percentages

Let's use our knowledge of included percentages in order to solve this question. We know that 2,950 yen resulted in \$26 after a 4% tax was charged. So let's start by calculating how much Mia would have gotten without the 4% tax.

$$1.04x = \$26$$

$$x = \$25$$

If there was no 4% tax, then Mia would have received \$25. Given this information, let's calculate many yen are in 1 dollar.

$$\frac{2{,}950 \text{ yen}}{\$25}$$

$$118 \text{ yen per dollar}$$

The exchange rate in yens per dollar is 118.

2 – Fill in Answer

Fill in "118" into the free response grid-in answer sheet.

2

A bank in Japan sells a prepaid credit card worth 3,750 yen. Mia can buy the prepaid card using dollars at the daily exchange rate with no fee, but she will lose any money left unspent on the prepaid card. What is the least number of the 3,750 yen on the prepaid card Mia must spend for the prepaid card to be cheaper than changing the money in the bank? Round your answer to the nearest whole number of yen.

Solution

1 – Ace Ratios & Percentages

Let's use our knowledge of included percentages in order to solve this question.

$$1.04x = 3{,}750 \text{ yen}$$

$$x = 3{,}605.7 \text{ yen}$$

This means that Mia should spend at least 3,605.7 yen for the prepaid card to be worth it. If she only spend 3,600 yen, then she would have been better off simply paying the 4% fee (a total of 3,744 yen) because she would have spent less than the 3,750 yen she spent on the prepaid card. However, if she spends 3,610 yen, then she is better off buying the prepaid card because paying the 4% fee would be more expensive (3,754.4 yen) than the 3,750 yen she paid for the prepaid card.

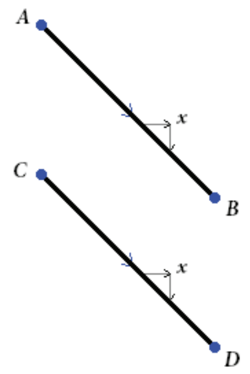
2 – Fill in Answer

Fill in "3606" into the free response grid-in answer sheet.

Ace Lines & Angles 15

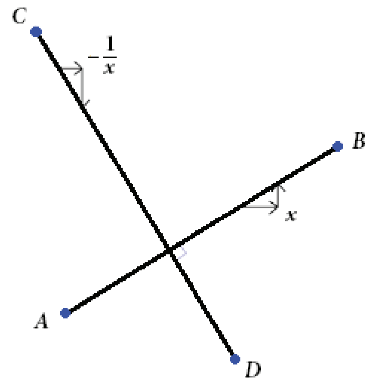
Parallel Lines

- Parallel lines never intersect
- The slopes of parallel lines are equal.



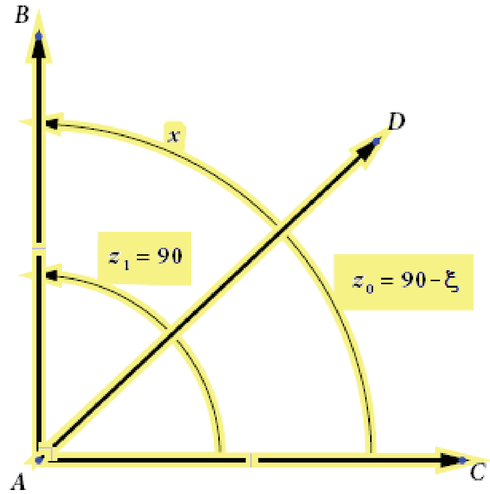
Perpendicular Lines

- Perpendicular lines form a 90° angle.
- The slopes of perpendicular lines are opposite reciprocals of one another.

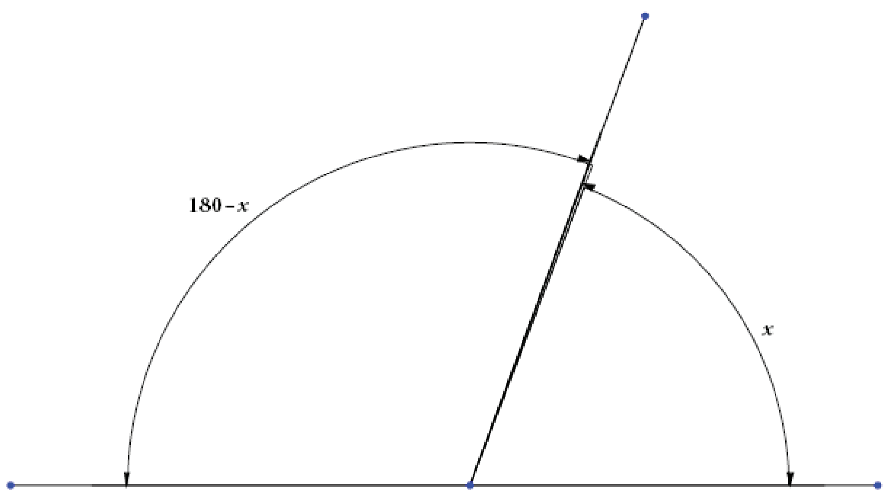


Angles

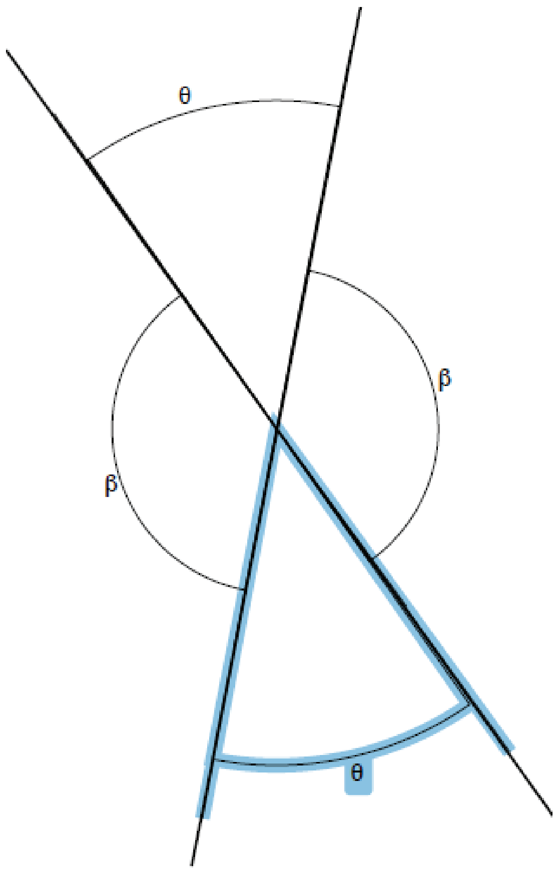
→ **Complementary angles form a 90° angle.**



→ **Supplementary angles form a 180° angle.**

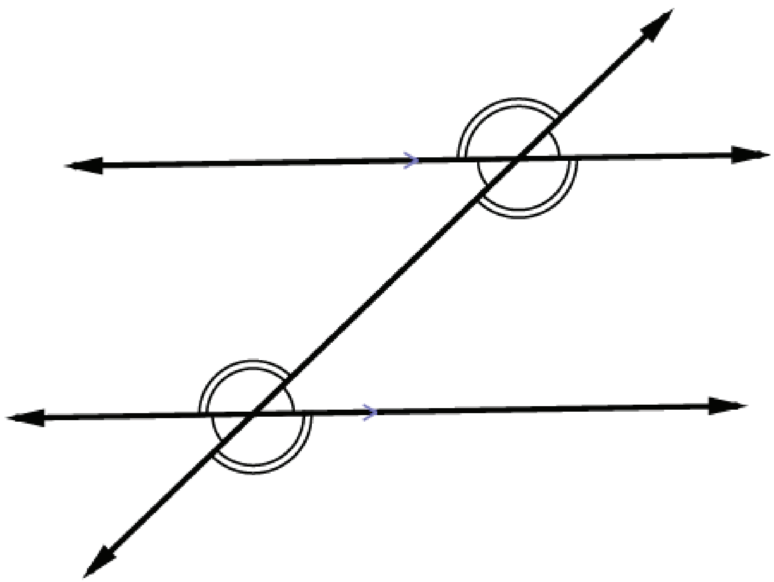


→ **Vertical angles are equal to each other.**

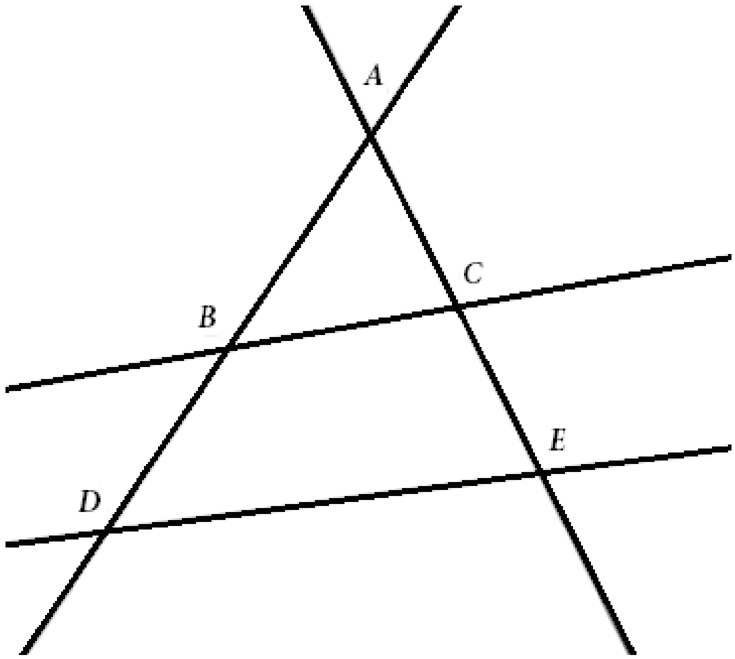


Transversal Through Parallel Lines

- Corresponding Angles are congruent
- Alternate Interior Angles are congruent
- Alternate Exterior Angles are congruent



Expert Practice



Note: Figure Not Drawn to Scale

1

In the figure above ΔABC is similar to ΔADE. Which of the following must be true?

(A) $BC \perp DE$

(B) $BC \parallel DE$

(C) $AD \perp AE$

(D) $AD \parallel AE$

Solution

1 – Ace Lines & Angles

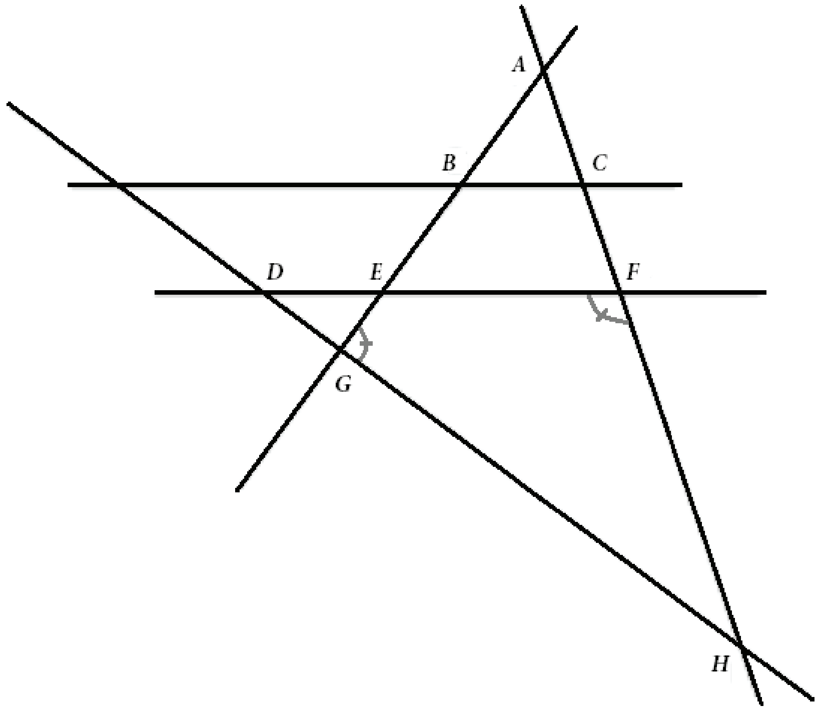
Although we haven't covered triangles yet, you should know that similar triangles have at least two angle measures that are congruent (their side measures are also proportional).

In the figure above, $\angle A$ is already common to both $\triangle ABC$ and $\triangle ADE$. Therefore, in order for the two triangles to be similar, $\angle B$ has to be equal to $\angle D$ or $\angle C$ has to be equal to $\angle E$. However, this would only work if the two lines BC and DE are parallel.

Corresponding angles are congruent angles that are formed when a line passes through two parallel lines. Hence, if $\angle B = \angle D$, they would be corresponding angles (the same is true for $\angle C = \angle E$). Given this information, we know that lines BC and DE must be parallel.

2 – Select Answer

Select answer choice B.



2

In the figure below ∠EFH = ∠EGH and BC || DF. Which of the following must be true?

(A) $AH \perp BC$

(B) $\angle EFH = \angle DEB$

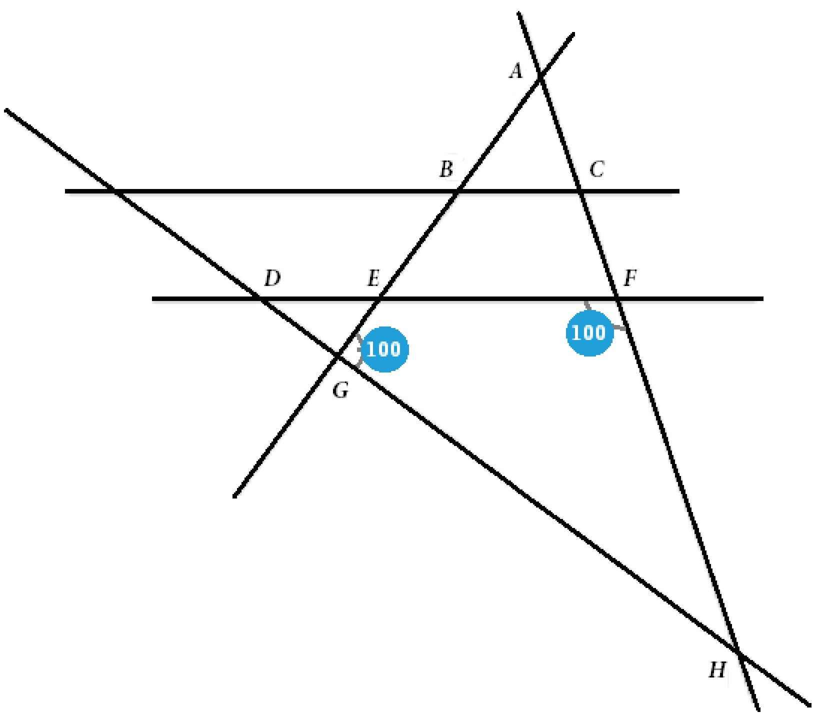
(C) $\Delta ABC \sim \Delta EDG$

(D) $\Delta ABC = \Delta EDG$

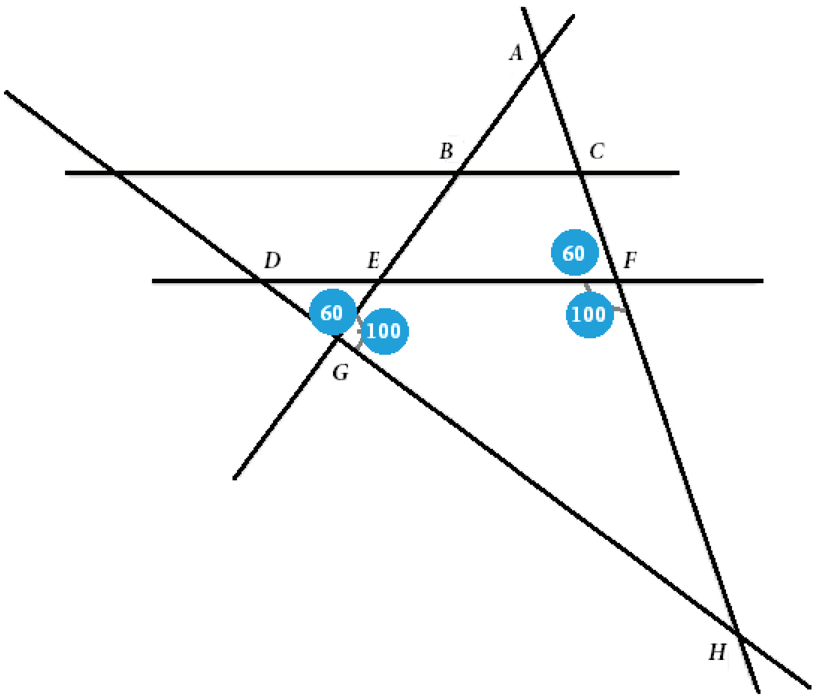
Solution

1 – Ace Lines & Angles

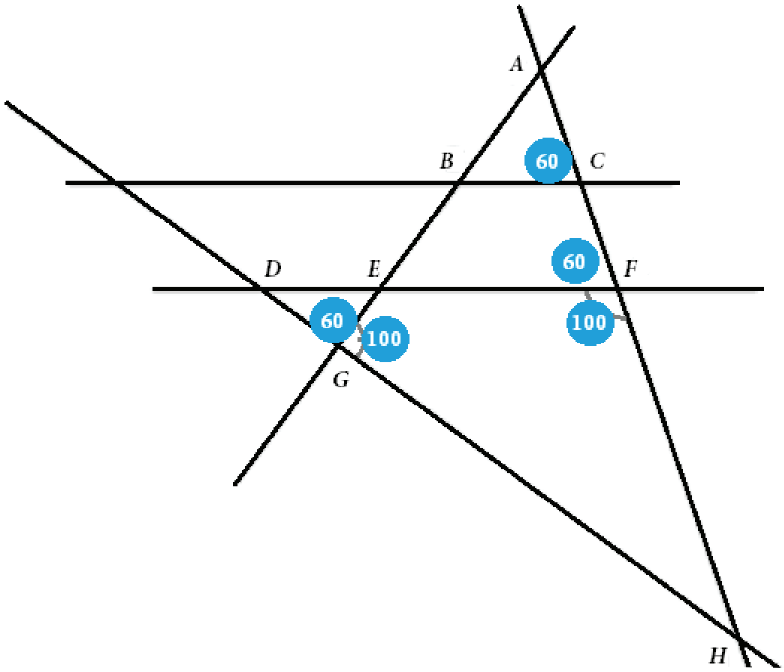
Using Math Expert Strategy 1 Substitute Abstracts with Tangibles, we should attempt to solve this problem my substituting our own angle measures in. I"ll start by plugging in 100° for the angle measures that are congruent. Note that I am purposely choosing 100° because the angles are larger than 90°.



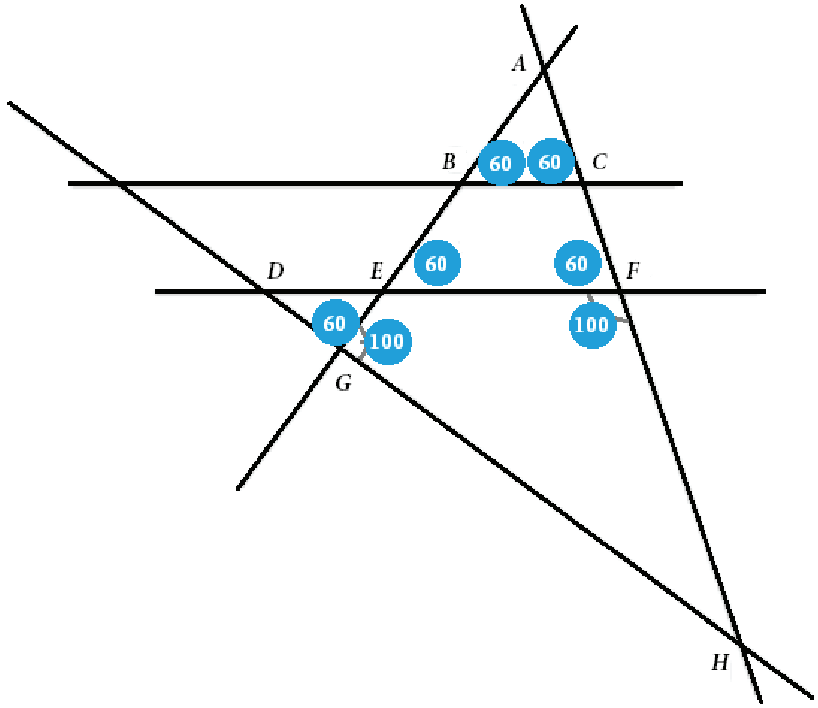
Using our knowledge of supplementary angles, we can fill in a couple more angles.



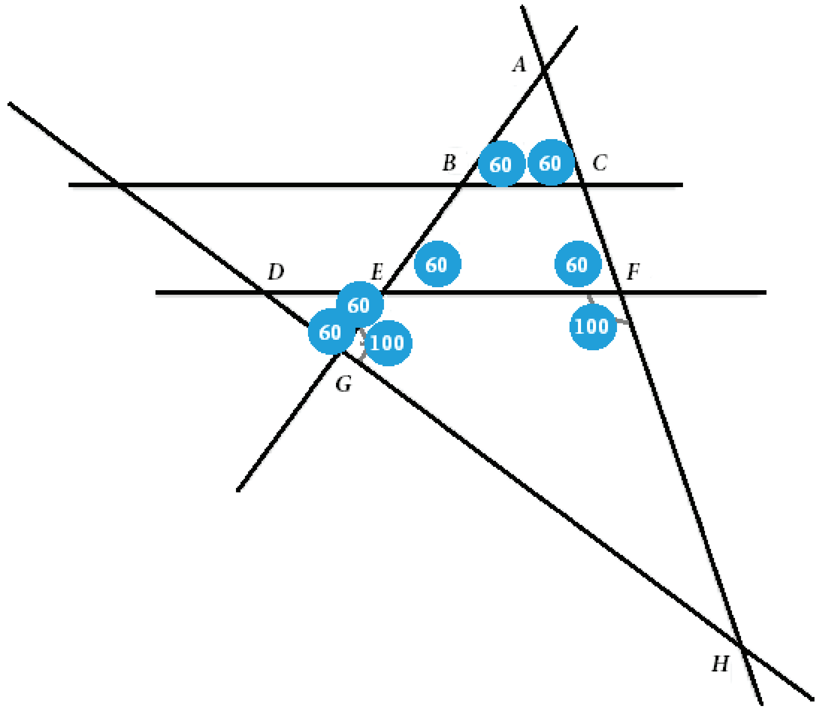
Using our knowledge of corresponding angles, we can fill in another angle.



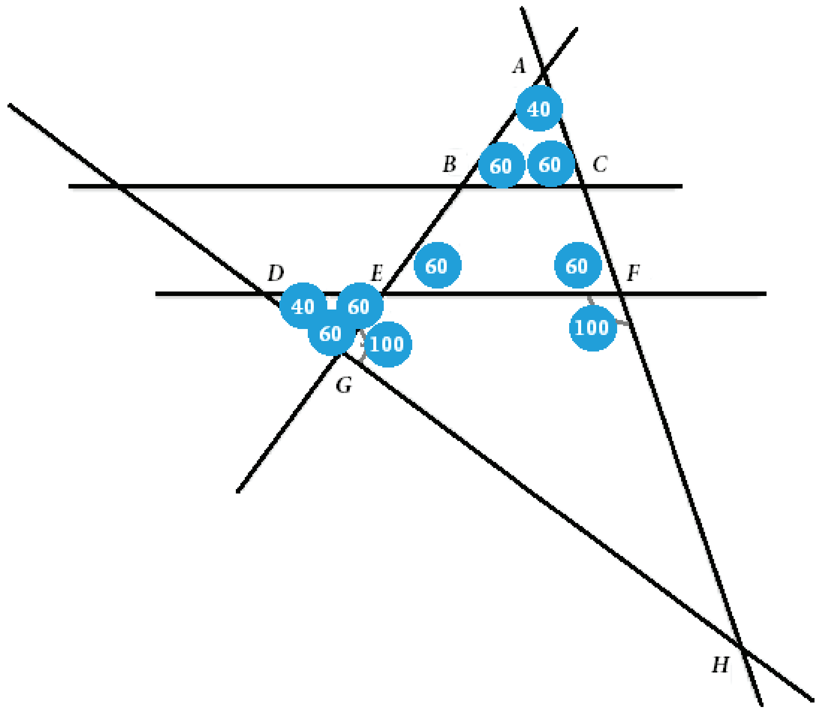
Using our knowledge of corresponding angles, we can fill in angle measures for ∠BEF and ∠ABC. Although there is no angle measure given to us, we can simply make one up using Math Expert Strategy 1 Substitute Abstract with Tangibles. I will plug in 60°.



Using vertical angles, we can also plug in 60° for ∠DEG.



Because all angles in a triangle add up to 180°, we can fill in 40° for the remaining angles in <ABC and <EDG.

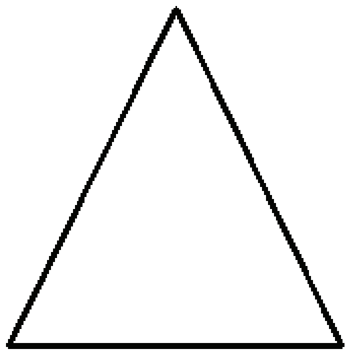


Now it's clear that all of the angles in <ABC and <EDG are equivalent. Does this make the two triangles similar or equivalent? Similar. Similar triangles have at least two angles that are equivalent. Equivalent triangles have all angle measures and side lengths congruent. In this case, we have no idea whether the side lengths of <ABC and <EDG are equal to each other. Therefore, <ABC and <EDG are similar.

2 – Select Answer

Select answer choice C.

Ace Triangles 16



Area

→ **The area of a triangle is equal to $\frac{1}{2}bh$. If the base of the above triangle is 6 and the height is 8,**

the area would be:

$$ü = \frac{1}{2}$$

$$A = \frac{1}{2}(6)(8)$$

$$A = 24$$

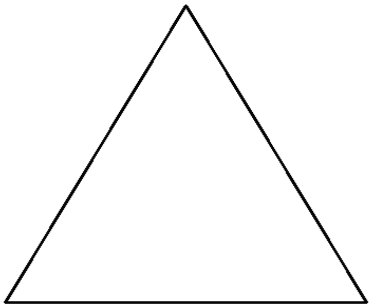
Interior Angles

→ **The interior angles of a triangle add up to 180°. If you know that two angles in a triangle are equal to 70°, you can easily calculate the remaining angle measure, x.**

$$180° = 70° + 70° + x°$$

$$40° = x°$$

Equilateral Triangles



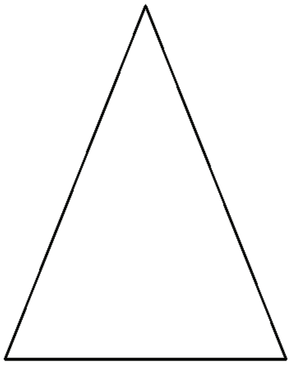
Angles

→ **All angles in an equilateral triangle are equivalent and equal to 60°.**

Side Lengths

→ **All side lengths in an equilateral triangle are equivalent.**

Isosceles Triangles



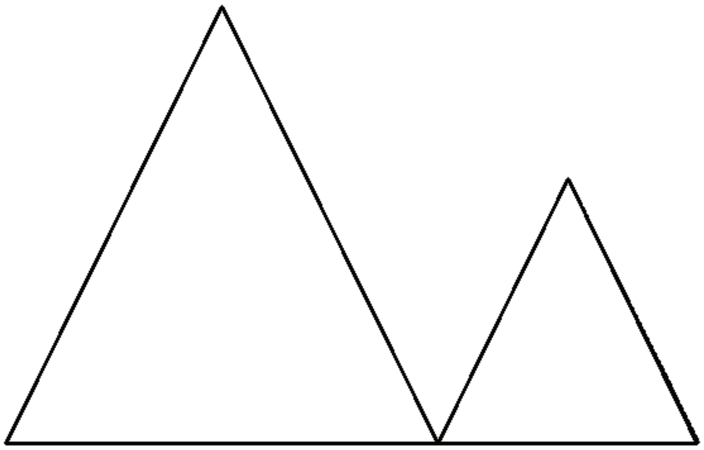
Angles

→ **Two angles in an isosceles triangle are equivalent.**

Side Lengths

→ **Two side lengths in an isosceles triangle are equivalent.**

Similar Triangles



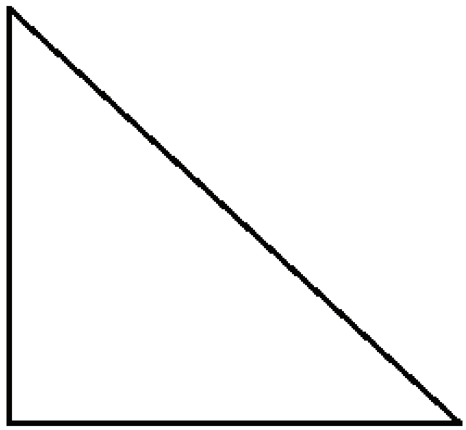
Angles

→ **All three angles in similar triangles are equivalent.**

Side Lengths

→ **All three pairs of side lengths in similar triangles are proportional.**

Right Triangles



Angles

→ **A right triangle has one 90° angle.**

Pythagorean Theorem

→ **If you know the value of two side lengths of a right triangle, you can figure out the any other side length of the triangle using the Pythagorean theorem (where c represents the hypotenuse). For example, let's assume that we know that the hypotenuse of a right triangle is 13 and that one side length is 5, what is the remaining side length?**

$$ü^{ü} + \quad =$$

$$ü^{ü} + b \quad =$$

$$5^2 + b^2 = 169$$

$$25 + b^2 = 169$$

$$b^2 = ü$$

$$b = 12$$

Pythagorean Triples

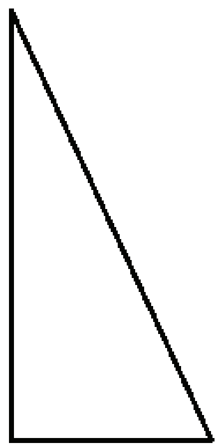
→ **Pythagorean triples are common side lengths of right triangles. You should be able to recognize that these side lengths quickly so that you don't have to go through the work of the Pythagorean formula. The first two numbers represent the side lengths and the last number represents the hypotenuse.**

3 – 4 – 5 (and any multiple of it i.e. 6 – 8 – 10 or 9 – 12 – 15)

5 – 12 – 13 (and any multiple of it i.e. 10 – 24 – 25)

Special Right Triangles

30-60-90 Triangles

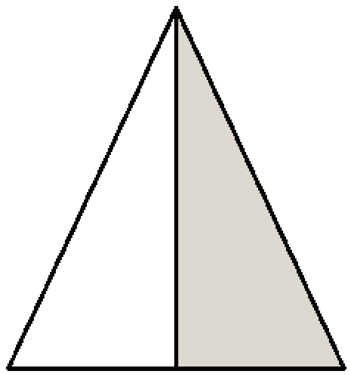


Side Lengths

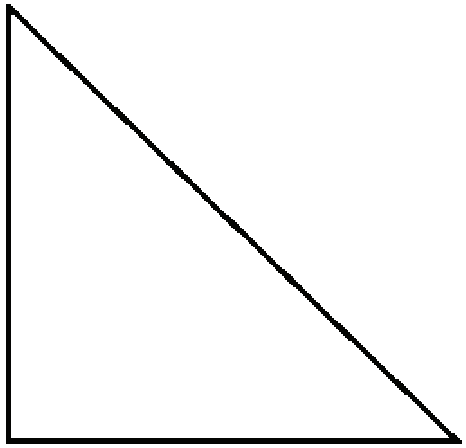
→ **You must know how the side lengths of a 30-60-90 triangle are related. The hypotenuse is double the short side of the triangle and the long side is $\sqrt{3}$ of the short side. The short side is the one opposite the 30° angle and the long side is the one opposite the 60° angle.**

Equilateral Triangles

→ **Whenever you see an equilateral triangle on the SAT, split it in half to form two 30-60-90 triangles. Doing so will often help you solve the question.**



45-45-90 Triangles

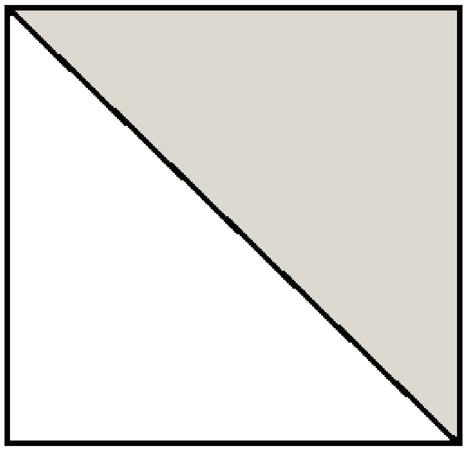


Side Lengths

→ **You must know how the side lengths of a 45-45-90 triangle are related. The hypotenuse is $\sqrt{2}$ of the side lengths. Both of the side lengths in a 45-45-90 triangle are of equal length.**

Squares

→ **Whenever you see a square on the SAT, draw a diagonal through it to form two 45-45-90 triangles. This means that the diagonal of a square will always be equal to $\sqrt{2}$ of the side length. Knowing this will often help you solve the question.**



Triangle Inequality Theorem

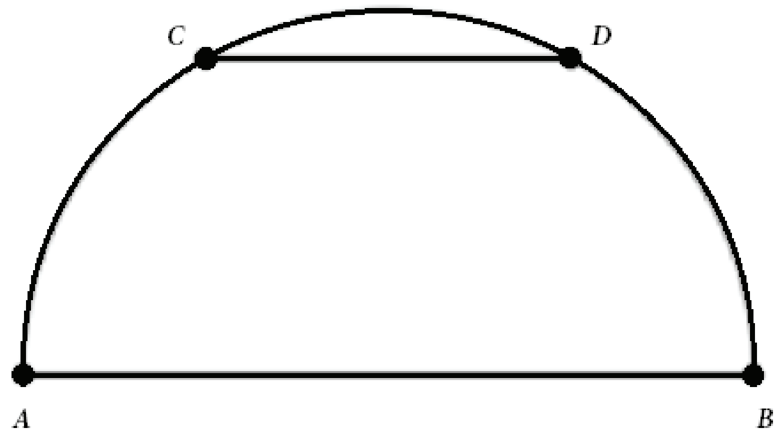
The third side of a triangle must be greater than the difference and less than the sum of the other two sides.

→ **Ex. Two sides of a triangle have the lengths 3 and 4. Which of the following could not be the perimeter of the triangle?**

(A) 9
(B) 11
(C) 13
(D) 15

→ **Answer D. The third side of the triangle must be less than the sum of the other two sides. In this case, the third side must be less than 7. Therefore, the perimeter could not be 15.**

Expert Practice



1

The semicircle above has a radius of r inches. Chord CD is parallel to the diameter AB. If the length of CD is equal to the radius of the semicircle, what is the distance between the chord and the diameter in terms of r ?

(A) $\frac{r\sqrt{3}}{2}$

(B) $\frac{r\sqrt{2}}{2}$

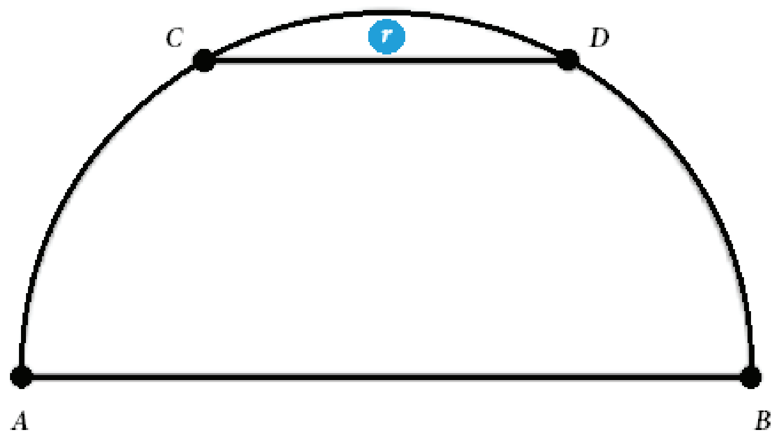
(C) r

(D) $\frac{2}{3}r$

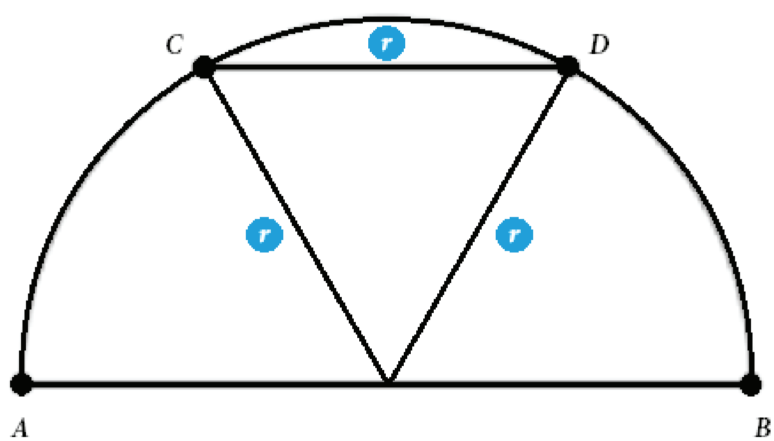
Solution

1 – Ace Triangles

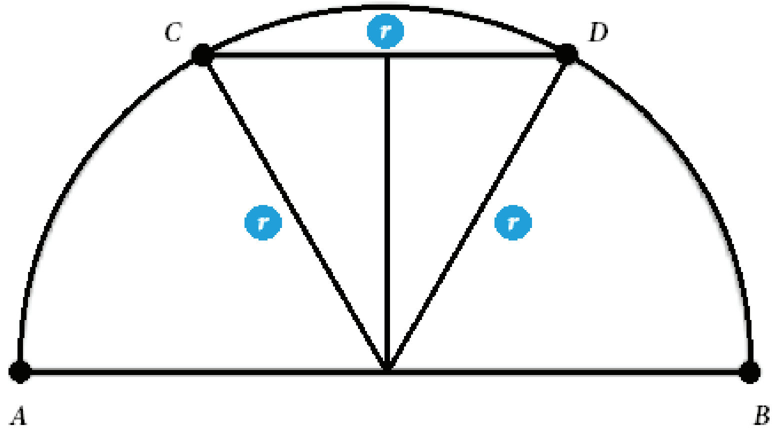
Although this question may seem like it's related to circles, it actually has more to do with triangles than anything else. Let's start by labeling the diagram.



The next key step is to draw in your own lines. One item that separates good SAT Math students from great SAT Math students is the ability to draw items on the diagram that the SAT doesn't give you directly. In this case, I would draw two radii on the circle, one from the center of the circle to point C and the other from the center of the circle to point D.



Now it should be clear that we have created an equilateral triangle. What did I say that you should always do to an equilateral triangle on the SAT? Split it in half to create two 30/60/90 triangles.

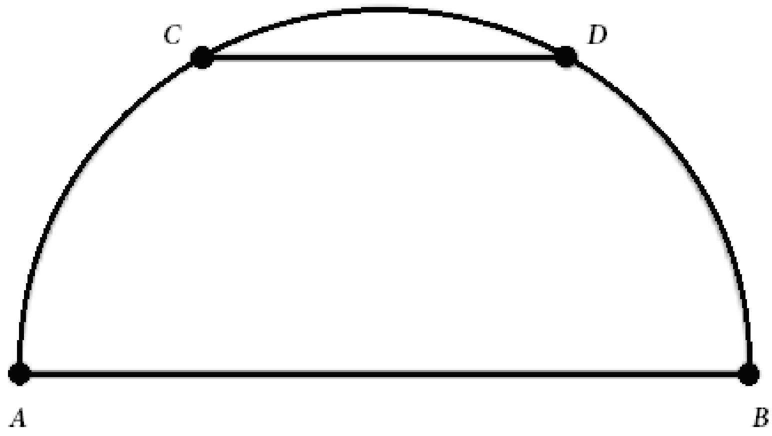


The long side length of these 30-60-90 triangles is going to be is $\sqrt{3}$ of the short side. The

short side in this case is $\frac{r}{2}$. Therefore, the distance between CD and AB is $\sqrt{3}\left(\frac{r}{2}\right)$.

2 – Select Answer

Select answer choice A.



2

The semicircle above has a radius of r inches. The distance between chord CD and diameter AB is $\frac{1}{3}$ of the length of AB. What is the area of the trapezoid $ABDC$ in terms of r ?

(A) $\frac{2}{3}r^2\left(1+\frac{\sqrt{8}}{3}\right)$

(B) $\frac{2}{3}r^2\left(1+\frac{\sqrt{5}}{3}\right)$

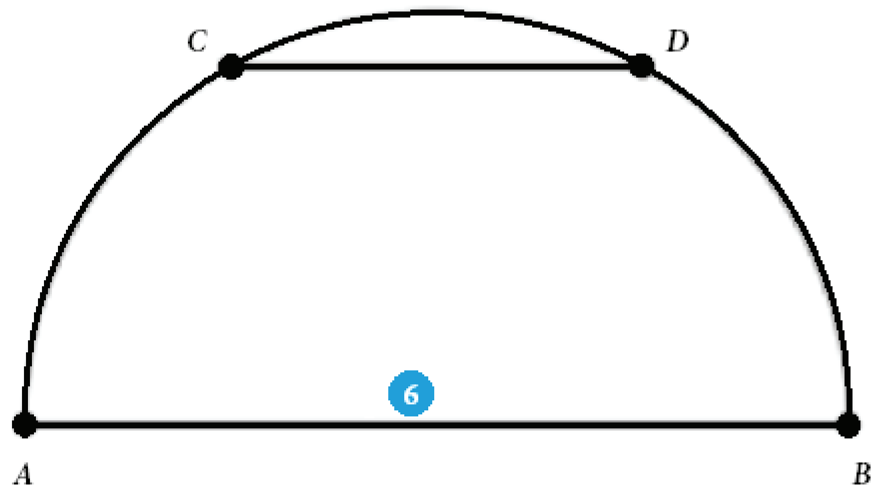
(C) $\frac{\pi r^2}{2}-\frac{\pi r^2}{2}$

(D) $\frac{2}{3}\pi r^2$

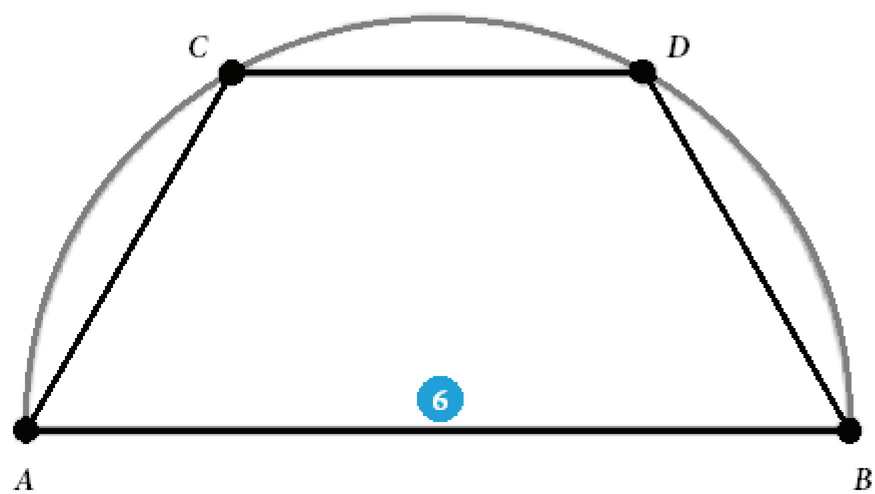
Solution

1 – Ace Triangles

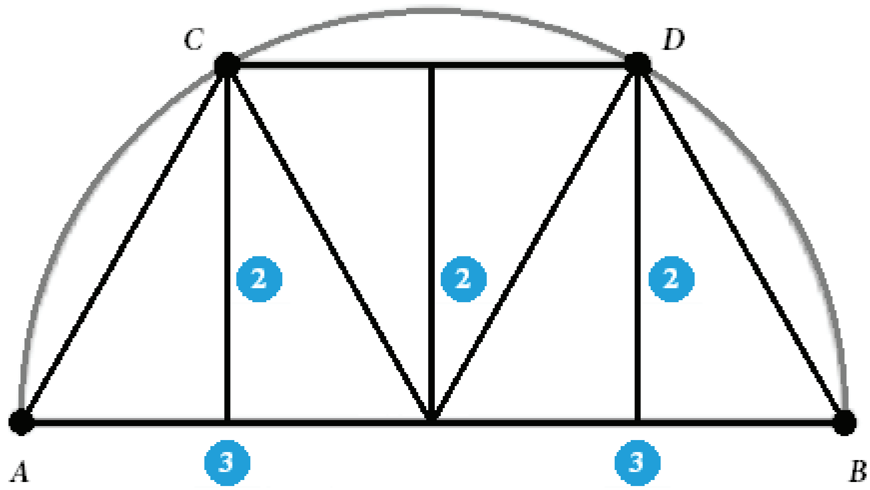
Let's solve this question using Math Expert Strategy #1 Substitute Abstract with Tangibles. First we should choose a length of AB that is easy to take $\frac{1}{3}$ of. Let's make AB 6.



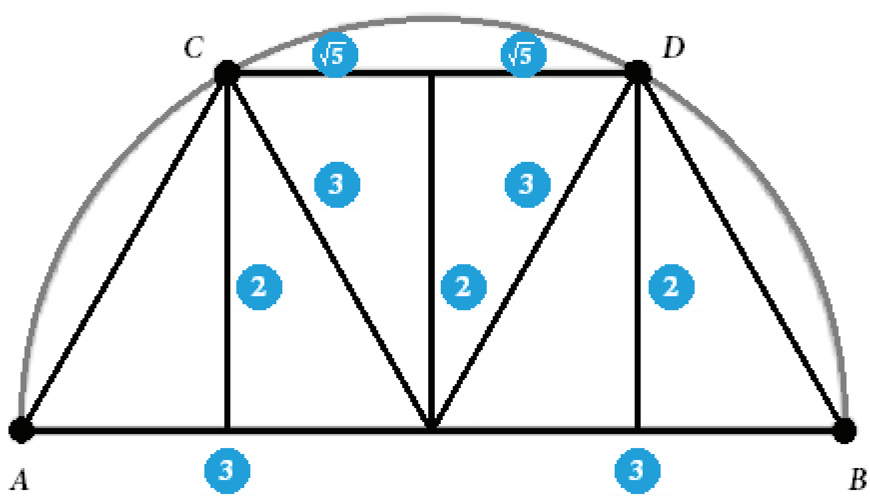
Now draw in the trapezoid.



Now, it's not necessary to know the formula for the area of a trapezoid. Instead, try to create triangles. Do this by drawing straight lines down from CD to AB and radii from the center to C and D.



As you can see, we have the base and height of every triangle, except the middle one. However, we can easily find it using Pythagorean's theorem.



Now, let's calculate the area of each triangle.

Left Triangle	→	$\frac{1}{2}(3)(2)$	=	**3**
Middle Triangle	→	$\frac{1}{2}(2\sqrt{5})(2)$	=	$2\sqrt{5}$
Right Triangle	→	$\frac{1}{2}(3)(2)$	=	**3**

Adding all of these areas up, the trapezoid area would be $6+2\sqrt{5}$ **. Which one of the answer choices gives us this result when we plug in r = 3?**

2 – Select Answer

Select answer choice B.

$$\left(\frac{2}{3}\right)(9)\left(1+\frac{\sqrt{5}}{3}\right)$$

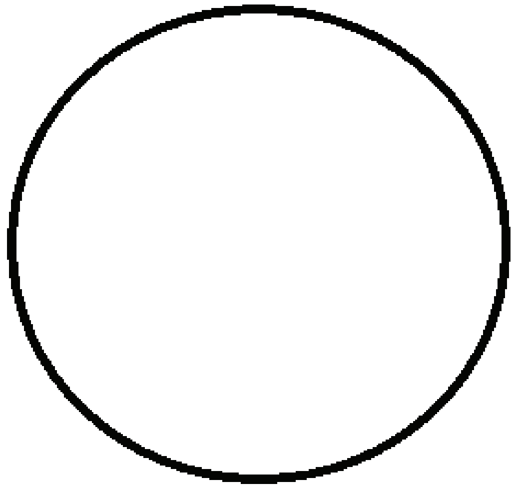
$$6\left(1+\frac{\sqrt{5}}{3}\right)$$

$$6+\frac{6\sqrt{5}}{3}$$

$$6+2\sqrt{5}$$

Notice how we did not need to know anything about trapezoids to solve this problem! When possible, use triangles.

Ace Circles 17



Radius

→ The distance from the center to any point on the circumference of the circle.

Diameter

→ The distance from one point on the circle to another point on the circle, but you must go through the center of the circle.

→ Double the radius

Area

→ πr^2

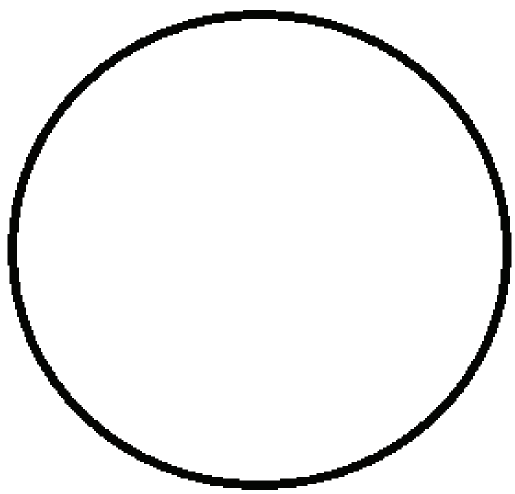
→ For example, if the radius of a circle is 3, then the area is 9π.

Circumference

→ $2\pi r$

→ For example, if the radius of a circle is 3, then the area is 6π.

Arcs & Sectors



Arc

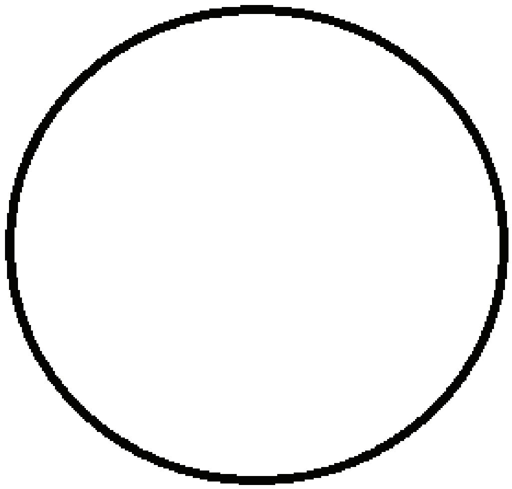
→ The distance from one point to another along the circumference of a circle.

Sector

→ The area of a circle enclosed by two radii and an arc

* In order to remember the difference between arcs and sectors, I think of a circle as a pizza. An arc is the crust and a sector is a slice!

Degrees & Radians



Degrees

→ A circle is made up of 360°.

Radians

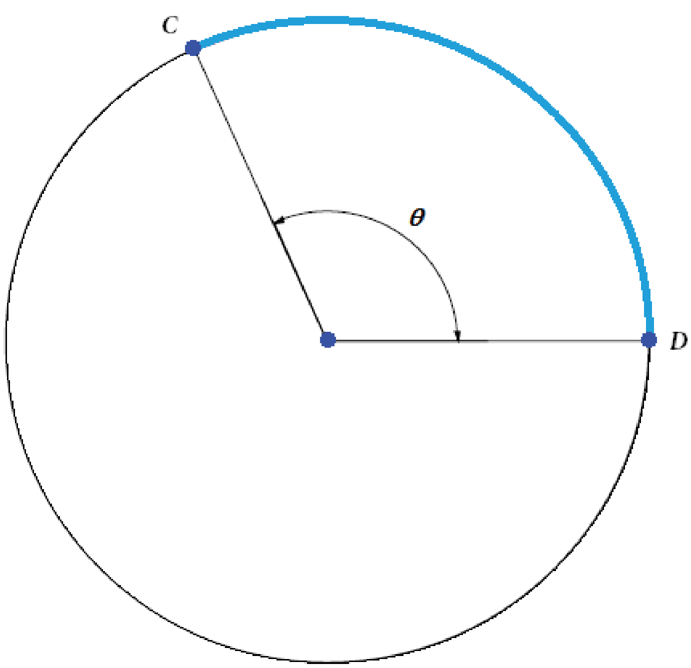
→ Radians can also express angle measures.

→ A circle is made up of 2π radians.

Arc Length & Central Angle Relationship

The central angle divided by 360° is equal to the arc length divided by the circumference, which is equal to area of a sector divided by the area of a circle.

→ **What is the arc length and sector area of CD if the central angle is 100° in a circle with a radius of 5?**



$$\frac{\text{Central Angle}}{360°} = \frac{\text{Arc Length}}{\text{Circumference}} = \frac{\text{Sector Area}}{\text{Area of Circle}}$$

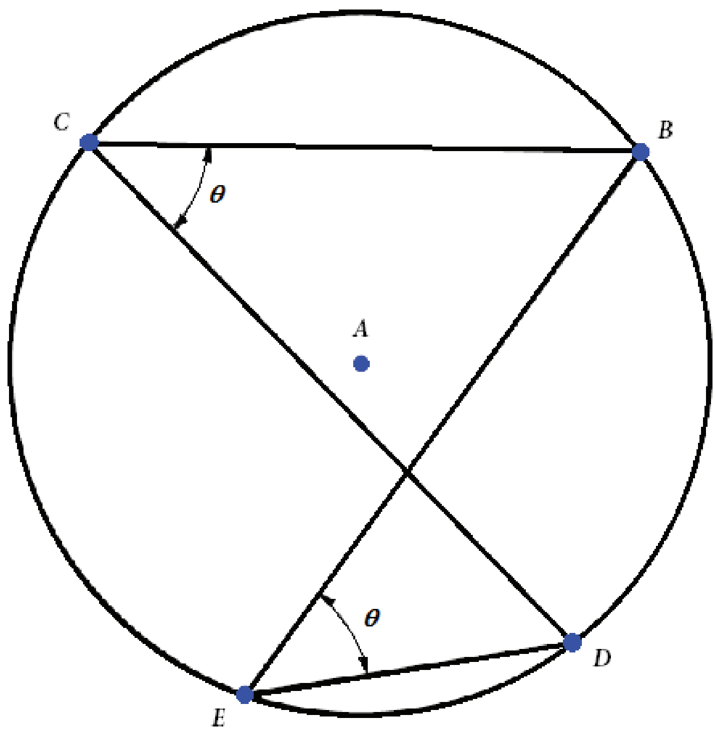
$$\frac{100°}{360°} = \frac{\text{Arc Length}}{10\pi} = \frac{\text{Sector Area}}{25\pi}$$

$$\frac{100°}{360°}(10\pi) = \text{Arc Length and Sector Area} = \frac{100°}{360°}(25\pi)$$

$$8.73 = \text{Arc Length and Sector Area} = 21.81$$

Inscribed Angles

- Inscribed angles are equal to $\frac{1}{2}$ of the angle measure of its intercepted arc.
- Inscribed angles that intercept the same arc are equal to each other.



- If the angle of arc BD is equal to 100°, then angle BCD is equal to 50°.
- Angle BCD and angle BED are equivalent because the intercept the same arc.

Equation of a Circle

$$(x-h)^2+(y-k)^2=r^2$$

(h, k) represents the center of the circle

Given an equation, you must able to complete the square in order to get the equation of the circle.

$$3x^2+3y^2-12x+24y-81=0$$

$$3\left(x^2+y^2-4x+8y\right)=81$$

$$3\left(x^2-4x+\_\_\right)+3\left(y^2+8y+\_\_\right)=81$$

$$3\left(x^2-4x+4\right)+3\left(y^2+8y+16\right)=81+12+48$$

$$3(x-2)^2+3(y+4)^2=141$$

$$(x-2)^2+(y+4)^2=47$$

The center of the circle is (2, –4) and the radius is $\sqrt{47}$.

Circle Equation Shifts

Circle graph shifts are different than traditional graph shifts in which you need to look at whether a number is added inside or outside of the parenthesis with x.

For example, for the circle below, let's shift the center down 2 units and to the right 3 units.

$$(x-2)^2+(y+4)^2=9$$

The current center is at (2, –4). Therefore, the new center would be at (5, –6).

$$(x-5)^2+(y+6)^2=9$$

Essentially, always do the opposite of what's intuitive for circle shifts. If moving up, subtract from the y. If moving left, add to the x.

Now, let's decrease the radius of the circle by 2. Currently, the radius is 3. To decrease the radius to 1, the new equation would look like the following:

$$(x-5)^2+(y+6)^2=1$$

Expert Practice

1

The equation of a circle in the *xy*-plane is shown below. What is the center of a circle?

$$x^2+y^2-2x+4y=16$$

(A) $(-1,-2)$
(B) $(1,-2)$
(C) $(-1,2)$
(D) $(1,2)$

Solution

1 – Ace Circles

Complete the Square.

$$x^2 + y^2 - 2x + 4y = 16$$

$$(x^2 - 2x + \_\_) + (y^2 + 4y + \_\_) = 16$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 16 + 1 + 4$$

$$(x - 1)^2 + (y + 2)^2 = 16 + 1 + 4$$

$$(x - 1)^2 + (y + 2)^2 = 21$$

The center of this circle is (1, –2).

2 – Select Answer

Select answer choice B.

2

The equation of a circle in the *xy*-plane is shown below. What is the distance between the center of the circle and the point (1,1)?

$$x^2 + y^2 + 8x - 4y = 124$$

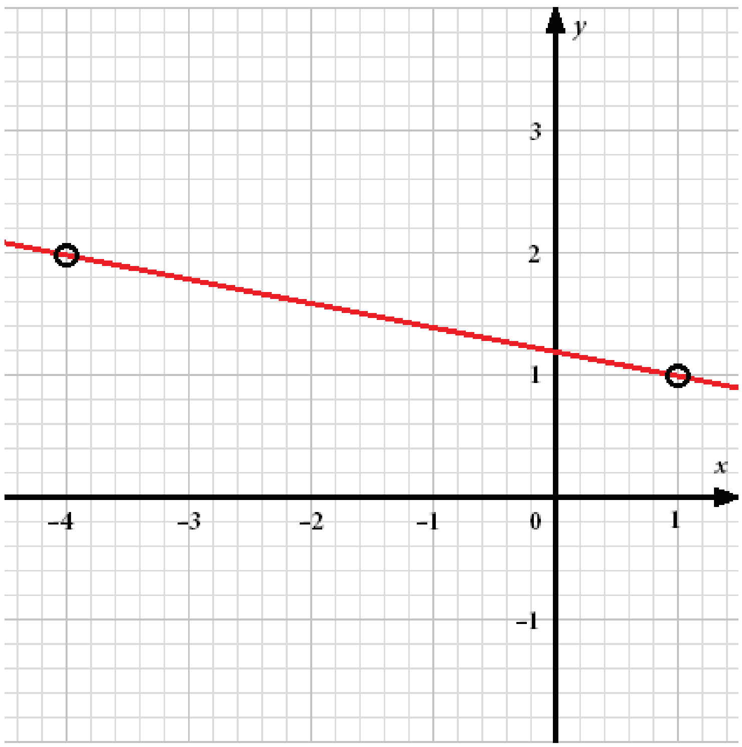
Solution

1 – Ace Circles

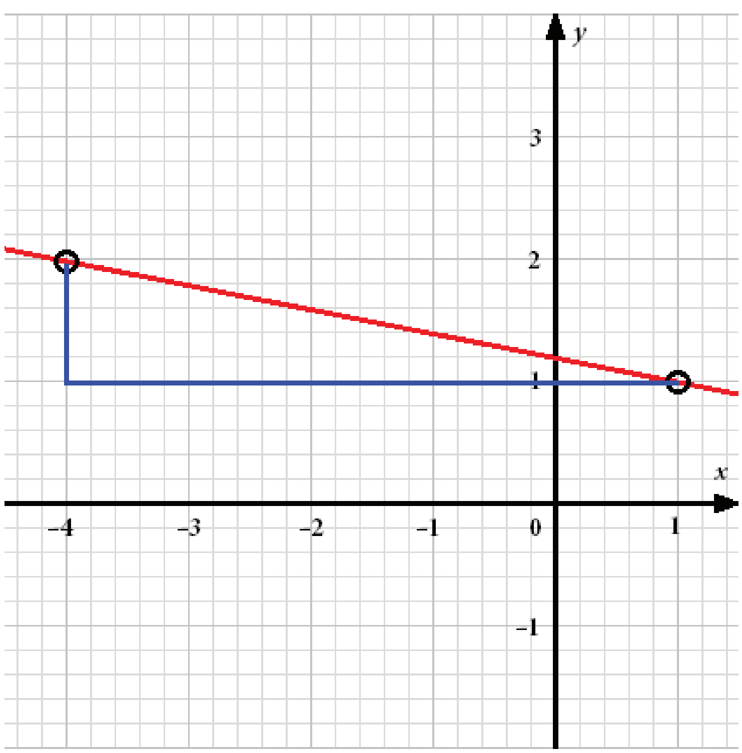
Complete the Square.

$$x^2 + y^2 + 8x - 4y = 124$$
$$(x^2 + 8x + \_\_) + (y^2 - 4y + \_\_) = 124$$
$$(x^2 + 8x + 16) + (y^2 - 4y + 4) = 124 + 16 + 4$$
$$(x+4)^2 + (y-2)^2 = 144$$
$$(x+4)^2 + (y-2)^2 = 12^2$$

The center of this circle is (–4, 2). Now we need to find the distance between this point and (1,1). You can do this in two ways. You can either remember the distance formula or you can use triangles. I prefer to use triangles, so I'll show you that method first. To use triangles to find the distance between two points, start by graphing where the points are on the coordinate plane.



Between any two points that are not on a perfectly horizontal or vertical line, you can always draw a right triangle. In this case, the side lengths of the right triangle are 1 & 2.



Using Pythagorean theorem, we can calculate the distance between the two points (the hypotenuse)

$$a^2 + b^2 = c^2$$
$$1^2 + 5^2 = c^2$$
$$26 = c^2$$
$$c = \sqrt{26}$$
$$x = 5.1$$

If you do not want to draw the points on the graph, you can also use the distance formula solve this problem.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(-4 - 1)^2 + (2 - 1)^2}$$
$$d = \sqrt{(-5)^2 + (1)^2}$$
$$d = \sqrt{25 + 1}$$
$$d = \sqrt{26}$$
$$d = 5.1$$

Both the Pythagorean theorem and the distance formula are essentially the same thing, but I prefer the Pythagorean theorem only because then I don't have to memorize the distance

formula.

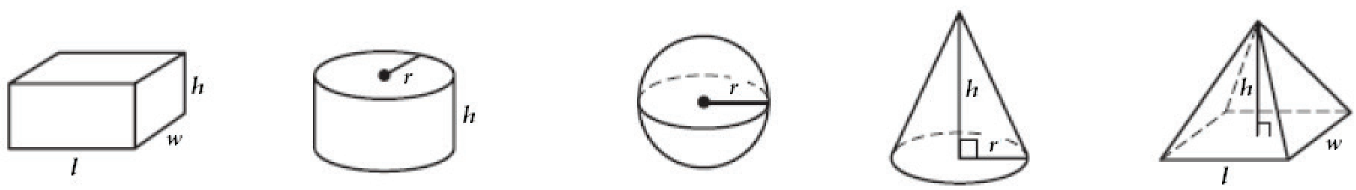
2 – Fill In Answer

Fill in 5.1 for the answer.

Ace 3D Figures 18

Standard Volumes

You must be familiar with the equations for the volume of each of the common 3D figures below. You do not need to memorize these formulas because they will be given to you at the beginning of each SAT Math section.



$V = lwh$ $\quad$ $V = \pi r^2 h$ $\quad$ $V = \frac{4}{3}\pi r^3$ $\quad$ $V = \frac{1}{3}\pi r^2 h$ $\quad$ $V = \frac{1}{3}lwh$

Custom Volumes

If the SAT asks you to find the volume of a 3D figure that is not a rectangular prism, a cylinder, a sphere, a cone, or a pyramid, then you must find the area of 1 face of the figure and multiply it by the height of the figure.

Volume vs. Mass vs. Density

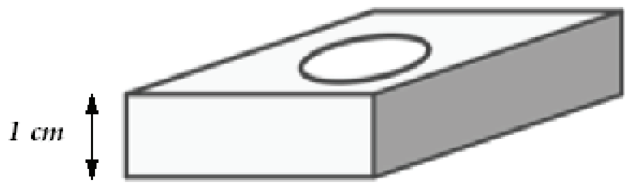
- Volume is the amount of space a 3D figure takes up.
- Mass is the amount of a 3D figure (for SAT purposes, you can think of it as weight)
- Density is mass/volume (weight per amount of space)

* To help you remember the difference among the 3, think of a balloon. A balloon has a low weight for the high amount of space it takes up. Therefore, it is has a low mass for it's high volume. In other words, a balloon has a low density.

Expert Practice

1

The figure below shows a metal square nut with two square faces and a thickness of 1 cm. The length of each side of a square face is 4 cm. A hole with a diameter of 2 cm is drilled through the nut. The density of the metal is 7.9 grams per cubic cm. What is the mass of this nut, to the nearest gram? (Density is mass divided by volume.)



Solution

1 – Ace 3D Figures

The unknown is mass. The density is given. Therefore, we should start by finding the volume.

→ **Volume of Rectangular Prism (Without Hole)**

$$4\ \text{cm} \times 4\ \text{cm} \times 1\ \text{cm}$$

$$16\ \text{cm}^3$$

→ **Volume of Hole (remember that radius is half the diameter)**

$$\pi r^2 h$$

$$\pi 1^2 1$$

$$\pi\ \text{cm}^3$$

→ **Volume of of Rectangular Prism (With Hole)**

$$16 - \pi \text{ cm}^3$$

$$12.86 \text{ cm}^3$$

Now that we have found the volume of the 3D figure, we can use density formula to figure out the mass.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\frac{7.9 \text{ grams}}{\text{cm}^3} = \frac{\text{Mass}}{12.86 \text{ cm}^3}$$

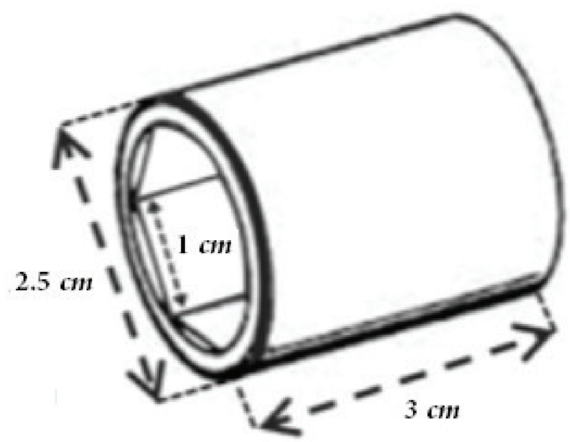
$$\text{Mass} = 101.6 \text{ grams}$$

2 – Fill in Answer

Fill in 102.

2

The figure below shows a metal nut socket with two circular faces and a thickness of 3 cm. The diameter of each circular face is 2.5 cm. A hole is drilled through the nut socket in such a way that a hexagonal prism is created inside with two regular hexagons as bases. The length of each side of a hexagonal base is 1 cm. The density of the metal is 11.3 grams per cubic cm. What is the mass of this nut socket, to the nearest gram? (Density is mass divided by volume.)



Solution

1 – Ace 3D Figures

The unknown is mass. The density is given. Therefore, we should start by finding the volume.

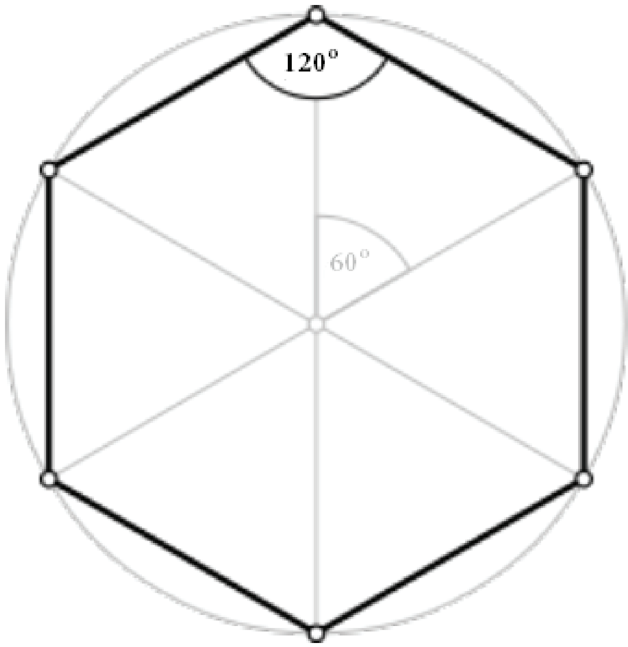
→ **Volume of Cylinder (Without Hexagon Cutout)**

$$\pi r^2 h$$

$$\pi 1.25^2 3$$

$$14.7\ \text{cm}^3$$

→ **Volume of of Hexagononal Prism**



When you draw a hexagon, you can see that it is made up of 6 equilateral triangles. Because we know that equilateral traingles can be split up into two 30-60-90 triangles, then a hexgon is actually made up of twelve 30-60-90 triangles.

We also know the base of each 30-60-90 triangle would be 0.5 (half of the 1cm hexagon side length). If the short side of each 30-60-90 triangle is 0.5, then the long side is $0.5\sqrt{3}$ **. The area of each 30-60-90 triangle is then:**

$$\frac{1}{2}(0.5)\left(0.5\sqrt{3}\right)$$

$$0.216\ \text{cm}^2$$

Multiplied by twelve, the area of the hexagonal face would be 2.6 cm^2. We can then multiply the area of this one face by the height to get the volume of the hexagonal prism.

$$(3\ \text{cm})\left(2.6\ \text{cm}^2\right)$$

$$7.8\ \text{cm}^3$$

→ **Volume of Cylinder (With Hexagon Cutout)**

$$14.7 \text{ cm}^3 - 7.8 \text{ cm}^3$$

$$6.9 \text{ cm}^3$$

Now that we have found the volume of the 3D figure, we can use density formula to figure out the mass.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\frac{11.3 \text{ grams}}{\text{cm}^3} = \frac{\text{Mass}}{6.9 \text{ cm}^3}$$

$$\text{Mass} = 77.97 \text{ grams}$$

2 – Fill In Answer

Fill in 77.97

Ace Complex Numbers 19

Definition

$i = \sqrt{-1}$

This is a new concept that the College Board is introducing to the SAT. Complex numbers, or imaginary numbers, are not "real." There is no such thing as the square root of –1. However, complex numbers are still important because the square of a complex number does exist.

$i^2 = -1$

The above is the most common item that you will see related to complex numbers on the SAT. You must remember that the square of i is equal to –1. Let's try an example expression.

$$(7-i)(4+3i)$$

$$28+21i-4i-3i^2$$

$$28+17i-3i^2$$

$$28+17i-3(-1)^2$$

$$28+17i-3(1)$$

$$28+17i-3$$

$$25+17i$$

Expert Practice

1

Which of the following is equal to the following expression? (Note: $i = \sqrt{-1}$)

$$\frac{1+i}{1-i}$$

(A) -1

(B) 1

(C) $-i$

(D) i

Solution

1 – Ace Complex Numbers

In order to try to get rid of the expression in the denominator, multiply both the top and bottom by (1 + i).

$$\frac{1+i}{1-i}$$

$$\frac{(1+i)(1+i)}{(1-i)(1+i)}$$

$$\frac{1+2i+i^2}{1-i^2}$$

$$\frac{1+2i+(-1)}{1-(-1)}$$

2 – Select Answer

$$\frac{2i}{2}$$

Select answer choice D.

$$i$$

2

Which of the following is equal to the following expression? (Note: $i = \sqrt{-1}$)

$$\frac{5-8i}{2+3i}$$

(A) $-\frac{14+31i}{13}$

(B) $-\frac{14-31i}{13}$

(C) $\frac{34-i}{13}$

(D) $\frac{14+31i}{5}$

Solution

1 – Ace Complex Numbers

In order to try to get rid of the expression in the denominator, multiply both the top and bottom by $(2-3i)$.

$$\frac{5-8i}{2+3i}$$

$$\frac{(5-8i)(2-3i)}{(2+3i)(2-3i)}$$

$$\frac{10-31i+24i^2}{4-9i^2}$$

$$\frac{10-31i+24(-1)}{4-9(-1)}$$

$$\frac{10-31i+\ -24}{13}$$

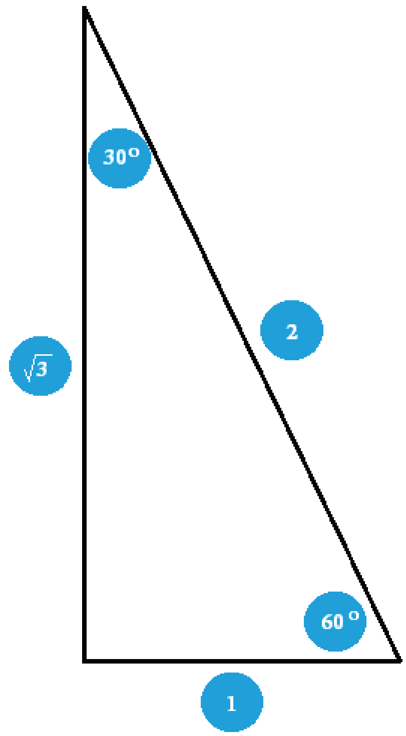
$$\frac{-14-31i}{13}$$

2 – Select Answer

$$-\frac{14+3i}{13}$$

Select answer choice A.

Ace Trigonometric Functions 20



Sine

$$\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}}$$

→ $\sin(30^\circ) = \frac{1}{2}$

→ $\sin(60^\circ)\frac{\sqrt{3}}{2}$

Cosine

$$\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

→ $\cos(30°) = \frac{\sqrt{3}}{2}$

→ $\cos(60°) = \frac{1}{2}$

*** I remember that "cosine" has the adjacent side in the numerator because your "coworker" is the person who sits adjacent to you (or next to you) at work. This makes it easy to remember. Then, "sine" is just the other one.**

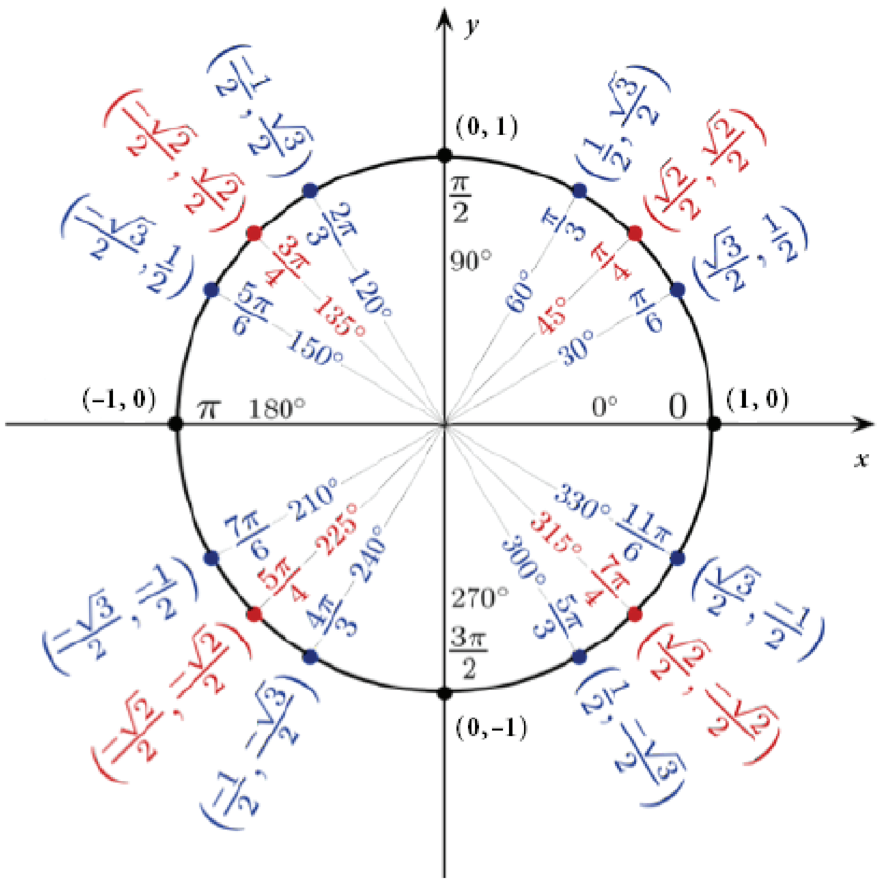
Tangent

$$\tan(x) = \frac{\text{opposite}}{\text{adjacent}}$$

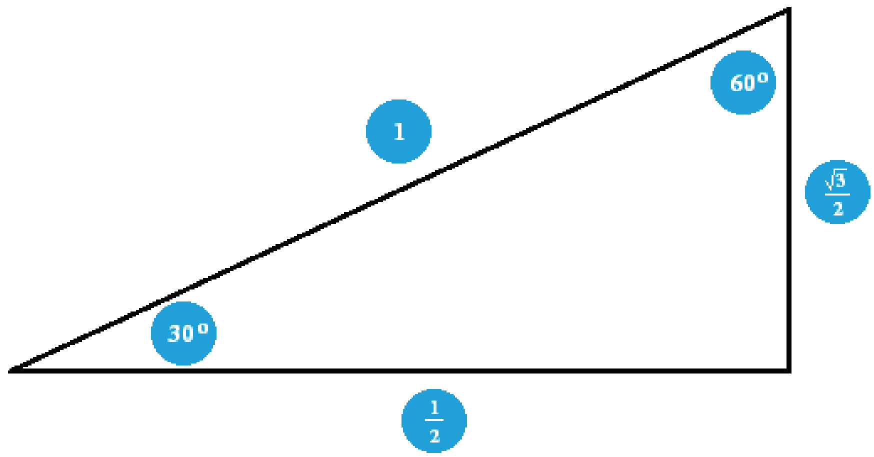
→ $\tan(30°) = \frac{1}{\sqrt{3}}$

→ $\tan(60°) = \frac{\sqrt{3}}{1}$

Degrees & Radians



If you can understand the above unit circle diagram, you will understand everything you need to know related to degrees and radians on the SAT. This is a diagram of a "unit circle," which is essentially a circle with a radius of 1. However, what's important to note are the triangles that each point on the unit-circle creates. For example, a triangle with a 30° angle would look like the following on the unit-circle:



- This is why the coordinates of the 30° (or $\frac{\pi}{6}$) angle on the unit-circle are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

- 2π radians = 360°

- To convert degrees to radians, multiply by $\frac{2\pi}{360°}$. For example, $30° \times \frac{2\pi}{360°} = \frac{\pi}{6}$.

- To convert radians to degrees, multiply by $\frac{360°}{2\pi}$. For example, $\frac{\pi}{6} \times \frac{360°}{2\pi} = 30°$.

- A trick that I use to quickly convert from radians to degrees is simply to think of π as 180°.

- Every right triangle you form on the unit circle will have a hypotenuse of 1. The angle measures will always be some multiple of 30°, 45°, and 60°. Therefore, you can use your knowledge of these triangles to figure out what the side lengths are. And the side lengths will tell you the coordinates on the graph of the unit-circle.

- The y-values on the unit-circle are equal to sine values because the hypotenuse is always 1 unit. The x-values on the unit-circle are equal to cosine values because the hypotenuse is always 1.

$\sin(x) = \sin(x \pm 2\pi)$

When the SAT asks you for an equivalent sine function to a sine function that you already have, add or subtract 2π radians (or 360°) to the angle.

→ **What is** $\sin\left(\frac{5\pi}{6}\right)$ **?**

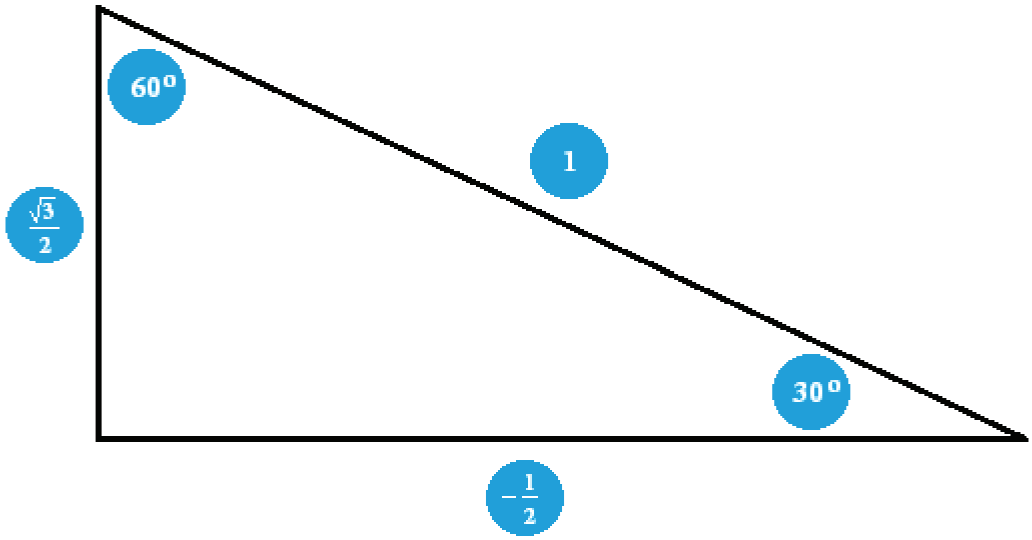
(1) Convert to Degrees

→ $\frac{5\pi}{6} \times \frac{360°}{2\pi} = 150°$

(2) Draw Triangle

*** Measure 150° counterclockwise from the x-axis. This would leave you 30° away from the xaxis.**

Always use angles as measured closest to the x-axis! Therefore, the triangle that you would use to solve this problem would be the following. You just have to be careful as to whether the x and y values will be negative or positive.



(3) Determine the sine of the angle.

→ $\sin(150°) = \dfrac{\frac{\sqrt{3}}{2}}{1} = \dfrac{\sqrt{3}}{2}$

→ **What is equivalent to** $\sin\left(\dfrac{5\pi}{6}\right)$ **?**

- $\sin\left(\dfrac{5\pi}{6} + 2\pi\right) = \sin\left(\dfrac{17\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$
- $\sin\left(\dfrac{5\pi}{6} - 2\pi\right) = \sin\left(-\dfrac{7\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$
- $\sin(150° + 360°) = \sin(510°) = \dfrac{\sqrt{\ }}{2}$
- $\sin(150° - 360°) = \sin(-210°) = \dfrac{\sqrt{3}}{2}$

*** You can continue adding or subtracting 2π or 360° to get equivalent sine values.**

In addition to adding and subtracting 2π, you should think about what other quadrant would have an equivalent y-value as your current sine value.

For example, $\sin(150°)$ is in the 2nd quadrant.

- **What other quadrant has positive y-values?**

  → **The 1st quadrant!**

- **What angle in the 1st quadrant would give you a positive y-value of $\dfrac{\sqrt{3}}{2}$?**

  → **30° (because it's 30° away from the x-axis)**

Therefore, $\sin(150°) = \sin(30°) = \dfrac{\sqrt{3}}{2}$**.**

$\cos(x) = \cos(x \pm 2\pi)$

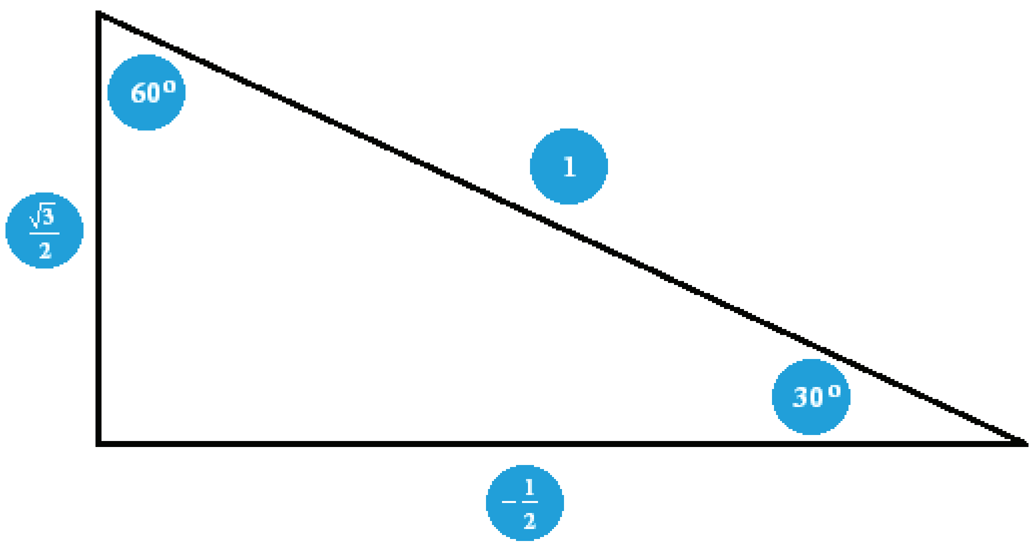
When the SAT asks you for an equivalent cosine function to a cosine function that you already have, add or subtract 2π radians (or 360°) to the angle.

→ **What is** $\cos\left(\frac{5\pi}{6}\right)$ **?**

(1) Convert to Degrees

→ $\frac{5\pi}{6} \times \frac{360°}{2\pi} = 150°$

(2) Draw Triangle



(3) Determine the cosine of the angle.

→ $\cos(150°) = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$

→ **What is equivalent to** $\cos\left(\frac{5\pi}{6}\right)$ **?**

- $\cos\left(\frac{5\pi}{6}+2\pi\right)=\cos\left(\frac{17\pi}{6}\right)=-\frac{1}{2}$
- $\cos\left(\frac{5\pi}{6}-2\pi\right)=\cos\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$
- $\cos(150^\circ+360^\circ)=\cos(510^\circ)=-\frac{1}{2}$
- $\cos(150^\circ-360^\circ)-\cos(-210^\circ)=-\frac{1}{2}$

* **You can continue adding or subtracting** 2π **or 360° to get equivalent cosine values.**

In addition to adding and subtracting 2π **, you should think about what other quadrant would have an equivalent x-value as your current cosine value.**

For example, $\cos(150^\circ)$ **is in the 2nd quadrant**

- **What other quadrant has negative x-values?**

  → **The 3rd quadrant!**

- **What angle in the 3rd quadrant would give you a negative x-value of** $-\frac{1}{2}$ **?**

  → **210° (because it's 30° away from the x-axis)**

Therefore, $\cos(150^\circ)=\cos(210^\circ)=-\frac{1}{2}$ **.**

$$\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$$

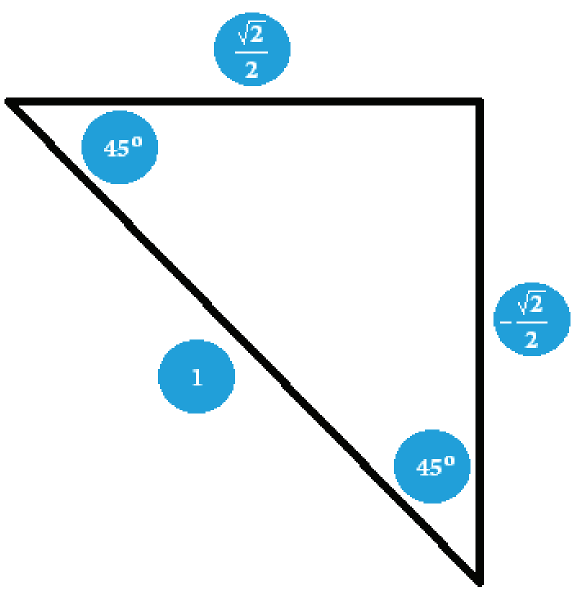
When the SAT asks you for an equivalent cosine function to a sine function that you already have, subtract the angle from $\frac{\pi}{2}$ radians (or 90°).

→ **What is** $\sin\left(\frac{7\pi}{4}\right)$ **?**

(1) Convert to Degrees

→ $\frac{7\pi}{4} \times \frac{360°}{2\pi} = 315°$

(2) Draw Triangle



(3) Determine the sine of the angle.

→ $\sin(315°) = \frac{-\frac{\sqrt{2}}{2}}{1} = -\frac{\sqrt{2}}{2}$

→ **What is equivalent to** $\sin\left(\frac{7\pi}{4}\right)$ **?**

- $\cos\left(\frac{\pi}{2}-\frac{7\pi}{4}\right)=\cos\left(-\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$
- $\cos(90°-315°)=\cos(-225°)=-\frac{\sqrt{2}}{2}$

* **You can continue adding or subtracting** 2π **or 360° to get equivalent cosine values.**

$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$$

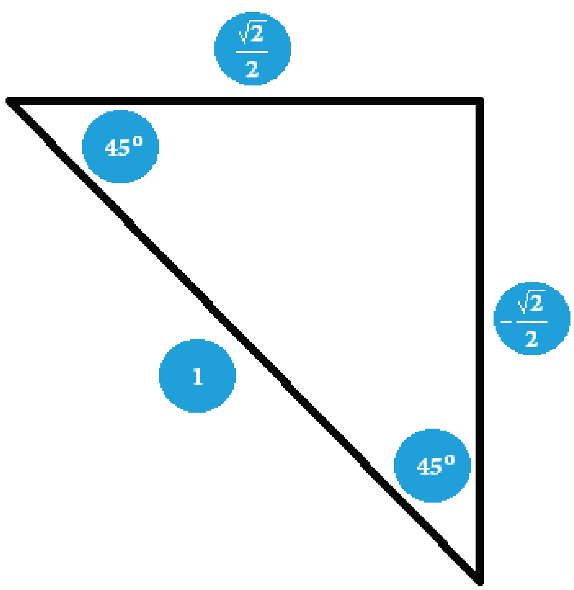
When the SAT asks you for an equivalent sine function to a cosine function that you already have, subtract the angle from $\frac{\pi}{2}$ radians (or 90°).

→ **What is** ü $\left(\frac{7\pi}{4}\right)$ **?**

(1) Convert to Degrees

→ $\frac{7\pi}{4} \times \frac{360°}{2\pi} = 315°$

(2) Draw Triangle



(3) Determine the cosine of the angle.

→ $\cos(315°) = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$

→ **What is equivalent to** ü $\left(\frac{7\pi}{4}\right)$ **?**

- $\sin\left(\frac{\aleph}{2}-\frac{\text{ü}}{4}\right)=\sin\left(-\frac{}{4}\right)=\frac{\sqrt{\ }}{2}$
- $\sin(90^\circ-315^\circ)=\sin(-225^\circ)=\frac{\sqrt{2}}{2}$

* **You can continue adding or subtracting** 2π **or 360° to get equivalent sine values.**

Complementary Angles: $\sin(A) = \cos(B)$

$A + B = 90°$

Since all you are doing to convert from sine to cosine is subtracting the angle from 90°, the two angles must be complementary. For example, what is x in the following problem?

$$\sin(x° + 20°) = \cos(2x° + 40°)$$

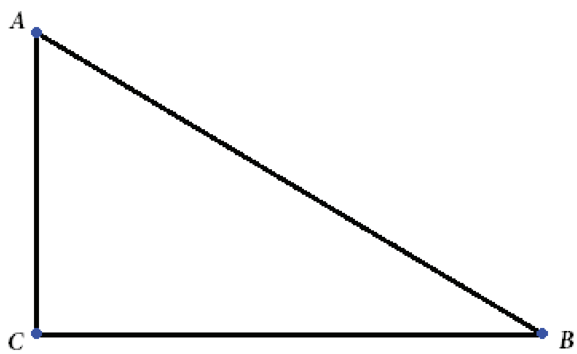
$$(x° + 20°) + (2x° + 40°) = 90°$$

$$3x° + 60° = 90°$$

$$\text{ü}x° = \quad °$$

$$x = 10°$$

In addition, if you know the sine of one angle in a 90° triangle, then the cosine of the other angle in a 90° triangle must be equivalent. For example, if sin(A) = 0.8 in the triangle below, what is cos(B)?



$$\cos(B) = 0.8$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

When the SAT asks you for the tangent of an angle that you know the sine and cosine values for, then you simply put sine over cosine.

→ **What is** $\tan\left(\frac{7\pi}{4}\right)$ **?**

(1) What is $\sin\left(\frac{\pi}{4}\right)$ **?**

→ $\frac{\sqrt{2}}{2}$

(2) What is $\cos\left(\frac{7\pi}{4}\right)$ **?**

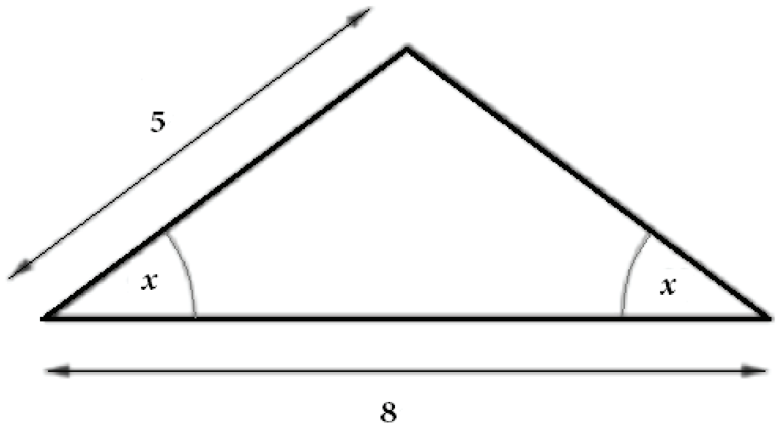
→ $-\frac{\sqrt{2}}{2}$

(3) $$\frac{\sin\left(\frac{7\pi}{4}\right)}{\cos\left(\frac{7\pi}{4}\right)}$$

→ $$\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

Expert Practice

1

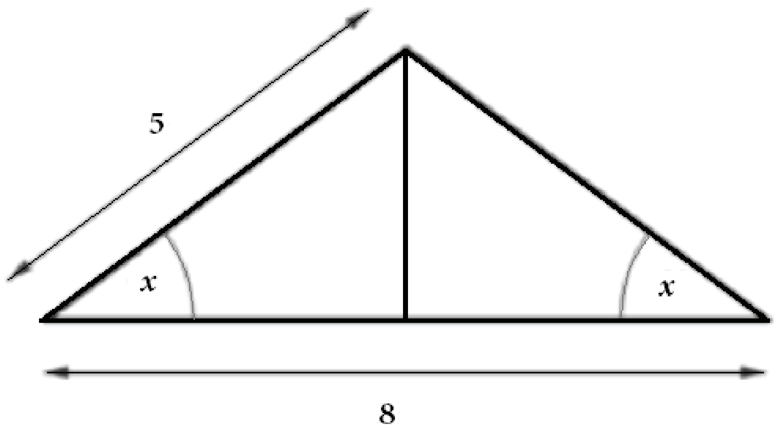


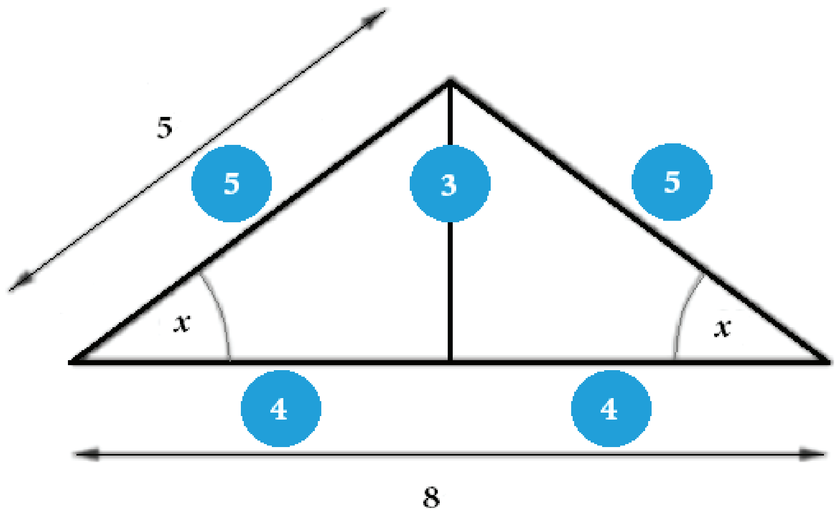
What is the value of $\sin(x)$ in the triangle above?

Solution

1 – Ace Trigonometric Functions

Remember that the difference between good SAT Math scores and great SAT Math scores is the ability to draw in lines on figures that are not already the diagram. So let's start by drawing the height of this triangle.





This creates two 3-4-5 right triangles. We know that the bottom base would be split exactly in half to 4 because the triangle is an isosceles triangle; therefore, the height bisects the opposite side exactly in half.

$$\sin(x) = \frac{3}{5}$$

2 – Fill in Answer

Fill in 3/5 or 0.60

2

Given that $\cos(x) = -a$, where x is a radian measure of an angle and $\frac{\pi}{2} < x < \frac{3\pi}{2}$. If $\cos(w) = a$, which of the following could be the value of w?

(A) $x +$

(B) $x - \frac{\pi}{2}$

(C) $x + 2\pi$

(D) $x - 2\pi$

Solution

1 – Ace Trigonometric Functions

We know that $\cos(x) = \cos(x \pm 2\pi)$ **. Therefore, we know the answer is not C or D because this question is asking about getting the opposite value. So let's try Substituting Abstracts with Tangibles in order to solve this problem.**

First, let's randomly choose select an angle. How about π **(or 180°)?**

Next, find ü (π) **.**

Since ü (π) **is just a straight line, it is just the x-value or –1.**

→ **– a = –1**

→ **a = 1**

→

What degree measure results in a $\cos(x) = 1$ **? I can think of only one place on the unit circle that the x-value is equal to 1. When the degree measure is 0° or 360°, then the** $\cos(x) = 1$ **. How can we get to a degree measure of 0° or 360° from the original degree measure of 180°? We would need to add or subtract 180° (or** π **).**

2 – Select Answer: **Select answer choice A.**

SAT MATH
PRACTICE

1

The scatter plot below shows the relationship between the work experience of 8 employees and their weekly salaries. The line of best fit is also shown.

Based on the line of best fit, what is the predicted weekly salary for an employee with 4 years of work experience?

(A) Between \$600 and \$650

(B) Between \$650 and \$700

(C) Between \$700 and \$750

(D) Between \$750 and \$800

Solution

1 – Ace Graphs

Whenever you're given a line graph, interpret one point. The first point says that when a person has 1 year of work experience, the expected weekly salary is $550.

Now determine what the expected weekly salary would be for an employee with 4 years of work experience based on the line of best fit. The line seems to hit between $700 and $750.

2 – Select Answer

Select answer choice C.

2

The table below shows how people of different ages spend their leisure time.

	Playing Sport	Watching TV	Reading Books	Total
Young	34	20	2	56
Middle Age	16	25	20	62
Older	1	18	26	45
Total	51	63	48	163

What fraction of all the people who watch TV or read books are young people?

Solution

1 – Ace Data Analysis

Whenever you're given a table, you should interpret one line. The first line says there are a total of 56 young people. 34 of the young people spend their leisure time playing a sport; 20 of the young people spend their leisure time watching TV; and 2 of the young people spend their leisure time reading books.

To answer the question, we must first determine the total number of people who watch TV or read books: 63 + 48 = 111.

Next, determine the number of young people who watch TV or read books: 20 + 2 = 22.

2 – Fill in Answer $\frac{22}{111}$

Because 22/111 is too long of an answer choice to fill into the grid-in spaces in your answer booklet, fill in the equivalent decimal value: .198

3

A survey was conducted among a randomly chosen sample of students about meals in the school. The table below displays a summary of the survey results.

Reported Voting By Grade				
	Voted	Did Not Vote	No Response	Total
5th Grade	72	30	23	125
6th Grade	71	35	25	131
7th Grade	68	34	18	120
8th Grade	69	31	16	116
Total	280	130	82	492

According to the table, for which grade did the greatest percentage of students report that they had voted?

(A) 5th Grade

(B) 6th Grade

(C) 7th Grade

(D) 8th Grade

Solution

1 – Ace Data Analysis

Whenever you're given a table, interpret one line. The first line says there are a total of 125 5th graders were surveyed in the sample. 72 of 5th graders voted on school meals; 30 of the 5th graders did not vote on school meals; and 23 of the 5th graders did not respond to the survey.

To answer the question, I would examine the fractions of voted over total for each grade level.

$$5^{th} \text{ Graders} \rightarrow \frac{72}{125}$$

$$6^{th} \text{ Graders} \rightarrow \frac{71}{131}$$

$$7^{th} \text{ Graders} \rightarrow \frac{68}{120}$$

$$8^{th} \text{ Graders} \rightarrow \frac{69}{116}$$

Without having to calculate the exact values, I can tell that the 6th grader voting percentage is lower percentage than the 5th grader voting percentage. Why? The numerator is smaller and the denominator is bigger.

Without having to calculate the exact values, I can tell that the 7th grader voting percentage is lower percentage than the 8th grader voting percentage. Why? The numerator is smaller and the denominator is bigger.

Now let's use our calculator to calculate the percentages of only 5th graders and 8th graders to determine which grade level had the highest voting participation.

5^{th} Graders → $\frac{72}{125}$ → $0.576 \times 100\%$ → 57.6%

8^{th} Graders → $\frac{69}{116}$ → $0.595 \times 100\%$ → 59.5%

2 – Select Answer

Select answer choice D.

4

A survey was conducted among a randomly chosen sample of students about meals in the school. The table below displays a summary of the survey results.

Of the 5^{th} grade students who reported voting, 20 were selected at random to do a follow-up survey where they were asked which meal they voted for. There were 5 candidates who voted for Meal A, and the other 15 voted for some other meal. Using the data from both the follow-up survey and the initial survey, which of the following is most likely to be an accurate statement?

Reported Voting By Grade				
	Voted	Did Not Vote	No Response	Total
5^{th} Grade	72	30	23	125
6^{th} Grade	71	35	25	131
7^{th} Grade	68	34	18	120
8^{th} Grade	69	31	16	116
Total	280	130	82	492

(A) About 5 fifth graders would report voting for Meal A in the survey.

(B) About 9 fifth graders would report voting for Meal A in the survey.

(C) About 18 fifth graders would report voting for Meal A in the survey.

(D) About 36 fifth graders would report voting for Meal A in the survey.

Solution

1 – Ace Ratios & Percentages

Given the sample information from the follow-up survey, calculate the percentage of the 5th graders that voted for meal A: $\frac{5}{20}$ **or 25%.**

Now apply that percentage to the fifth graders who voted in the initial survey: 25% of 72 = 18. We can extrapolate that 18 fifth graders from the ones surveyed voted for Meal A.

2 – Select Answer

Select answer choice C.

5

A researcher wanted to know if there is an association between cigarette smoking and cancer for the population of 50-year-olds in the United States. She obtained survey responses from a random sample of 3000 United States 50-year-olds and found convincing evidence of a positive association between cigarette smoking and cancer. Which of the following conclusions is well supported by the data?

(A) There is a positive association between cigarette smoking and cancer for 50-year-olds in the United States.

(B) There is a positive association between cigarette smoking and cancer for 50-year-olds in the world.

(C) Using cigarette smoking and cancer as defined by the study, an increase in cancer is caused by an increase of cigarette smoking for 50-year-olds in the United States.

(D) Using cigarette smoking and cancer as defined by the study, an increase in cancer is caused by an increase of cigarette smoking for 50-year-olds in the world.

Solution

1 – Ace Data Analysis

Remember the difference between correlation and causation. Correlation implies association. Causation implies one item produces the other.

Before looking at the answer choices, try to determine what the data statement means in your own words: For 50 year-olds in the United States, there is a positive correlation between cigarette smoking and cancer.

2 – Select Answer

Select answer choice A.

6

John pays his workers with tickets. Each ticket is worth $32. A worker earns $60 per hour. John has 24 tickets. Which of the following functions models the number of tickets remaining t hours after a worker has started his work?

(A) $f(t) = 24 - \frac{60t}{32}$

(B) $f(t) = 24 - \frac{32t}{60}$

(C) $f(t) = \frac{24 - 60t}{32}$

(D) $f(t) = \frac{24 - 32t}{60}$

Solution

1 – Substitute Abstracts with Tangibles

Let's say that a worker works for 2 hours.

In other words, t = 2.

The worker would then be owed $120.

The number of tickets needed to pay $120 would be $120 / $32 = 3.75 tickets.

The number of tickets remaining would be 24 – 3.75 = 20.25 tickets remaining.

Which function would give us 20.25 when we plug in t = 2?

(A) $f(t) = 24 - \frac{\text{ü}\left(\ \right)}{32}$ ✓

(B) $f(t) = 24 - \frac{\text{ü}\left(\ \right)}{60}$ ×

(C) $f(t) = \frac{24 - 60(2)}{32}$ ×

(D) $f(t) = \frac{24 - 32(2)}{60}$ ×

*** Note: To save time, you don't need to plug in every substitution into your calculator. Instead, you should be able to ballpark to get rid of some answers. For example 24 - 120 in answer choice C will clearly result a negative number.**

2 – Select Answer

Select answer choice A.

7

The price of a pen is \$7.80 and the price of a book is \$12.50. During a five-hour period, a total of 56 pens and books are sold, and the total earning was \$559. Solving which of the following systems of equations yields the number of pens, x, and the number of books, y, that were sold during the five hours?

(A) $\begin{cases} x + y = 56 \\ 12.5x + 7.8y = 559 \end{cases}$

(B) $\begin{cases} x + y = 559 \\ 12.5x + 7.8y = 56 \end{cases}$

(C) $\begin{cases} x + y = 56 \\ 7.8x + 12.5y = 559 \end{cases}$

(D) $\begin{cases} x + y = 559 \\ 12.5x + 7.8y = 56 \end{cases}$

Solution

1 – Ace Equations

Here we need to translate a word problem into a system of equations.

Let's start by creating the simpler equation first. We know that the pencils and books need to add up to 56. Therefore, x + y = 56.

Next, let's think about the dollars associated with this sale. We know that the price of a pen is $7.80, so $7.80 has to be multiplied by the number of pens (x). We also know that the price of a book is $12.50, so $12.50 has to be multiplied by the number of books (y). The overall equation that describes the total prices of pens and books sold together would appear as the following:

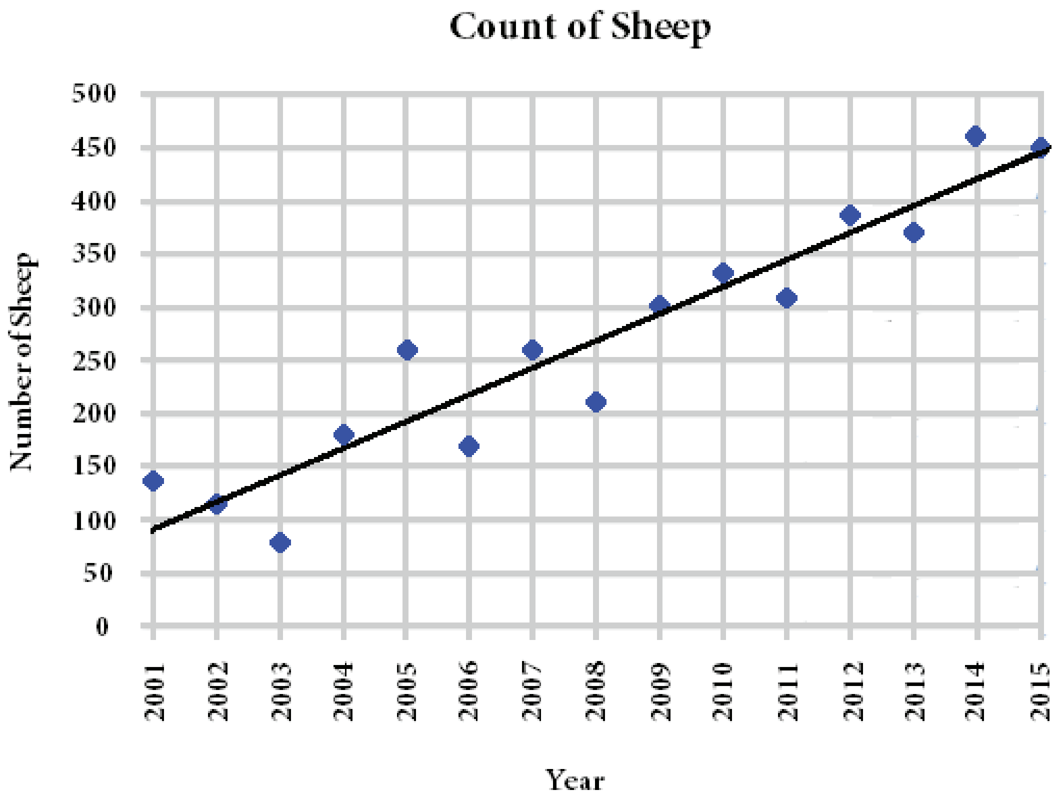
$$7.8x + 12.5y = 559$$

2 – Select Answer

Select answer choice C.

8

The scatter plot below shows counts of sheep on the farm from 2001 to 2015.



Based on the line of best fit to the data shown, which of the following values is closest to the average yearly increase in the number of sheep?

(A) 0.25

(B) 25

(C) 50

(D) 250

Solution

1 – Ace Graphs

Whenever you're given a line graph, interpret one point. The first point says that in 2001, there were just under 150 sheep.

To determine the average yearly increase based on the line of best fit, simply find the slope of the line. To do this, choose any two points on the line (I am approximating the y-values below):

(2001, 100)

(2003, 150)

Find Slope

$$\frac{150-100}{2003-2001}$$

$$\frac{50}{2}$$

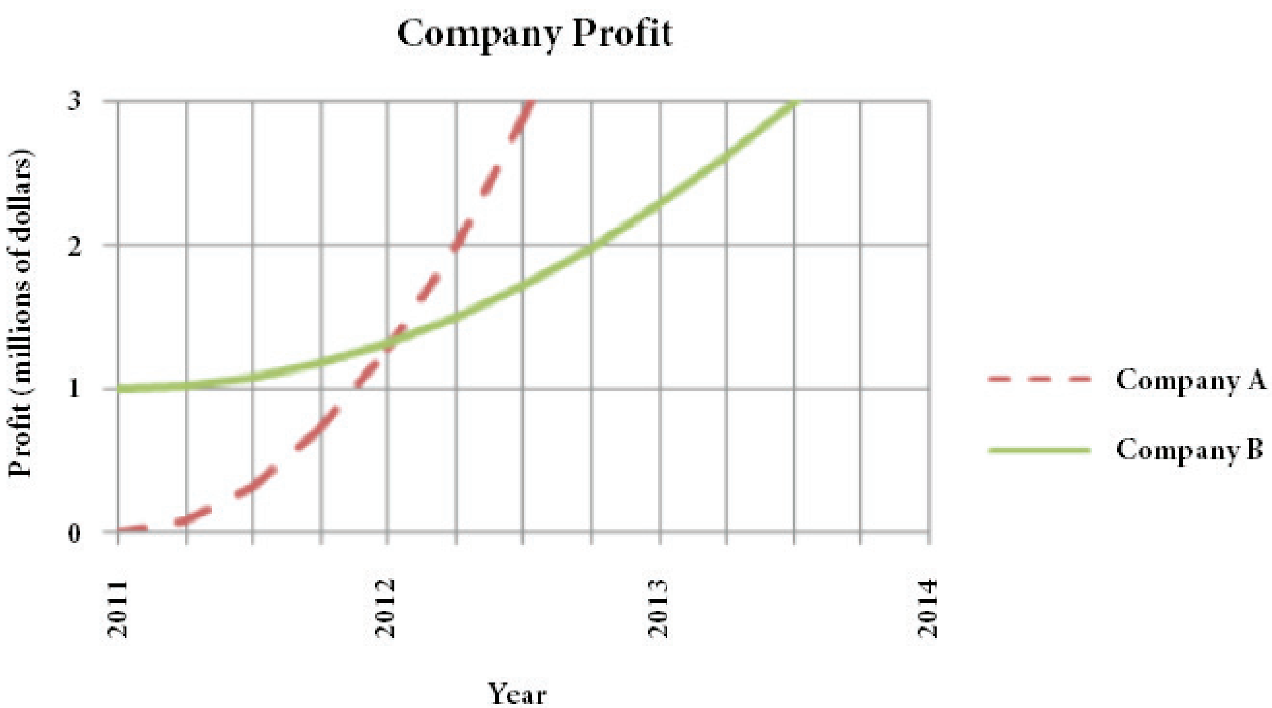
25

2 – Select Answer

Select answer choice B.

9

The graph below shows the increase in profits of Company A and Company B.



Which of the following is a correct statement about the data above?

(A) In 2011 both companies have had equal profits

(B) Companies have never had equal profits

(C) In the first year, the profit of Company A is increasing at a higher average rate than the profit of Company B

(D) In the first year, the profit of Company B is increasing at a higher average rate than the profit of Company A

Solution

1 – Ace Graphs

Whenever you're given a graph, interpret one point. The first point on the dashed line says that in 2011, Company A had a profit of $0. The first point on the solid line says that in 2011, Company B had a profit of $1 million.

Examine each answer choice.

(A) This is not true based on our interpretation of the first point on each line above.

(B) This is not true because the dashed line and the solid line cross in 2012 at a profit of about $1.2 million each.

(C) This is true because the positive slope of company A is steeper than the positive slope of company B between 2011 and 2012.

(D) This is not true because the positive slope of company B is less steep than the positive slope of company A between 2011 and 2012.

2 – Select Answer

Select answer choice C.

10

If (x, y) is a solution to the system of equations below, what is the value of y^2 ?

$$\begin{cases} x^2 - y^2 = 24 \\ x = -2y \end{cases}$$

(A) 3

(B) 8

(C) 12

(D) 24

Solution

1 – Substitute Answers in Problem (SAP)

You could certainly solve this system of equations using traditional high school algebra. However, I will show you how to use another strategy to solve systems of equations: Substitute Answers in Problem.

Let's start by assuming that answer choice is A is correct. Then, the following is true:

$$y^2 = 3$$

$$y = \sqrt{3}$$

$$x = -2\sqrt{3}$$

Now that we have potential x and y values, plug the numbers into the other equation to determine whether it works out.

$$x^2 - y^2 = 24$$

$$\left(-2\sqrt{3}\right)^2 - \sqrt{3}^2 = 24$$

$$(4 \cdot 3) - 3 = 24$$

$$9 \neq 24$$

Clearly, answer choice A is incorrect. Let's try answer choice B.

$$y^2 = 8$$

$$y = \sqrt{8}$$

$$x = -2\sqrt{8}$$

$$x^2 - y^2 = 24$$

$$\left(-2\sqrt{8}\right)^2 - \sqrt{8}^2 = 24$$

$$(4 \cdot 8) - 8 = 24$$

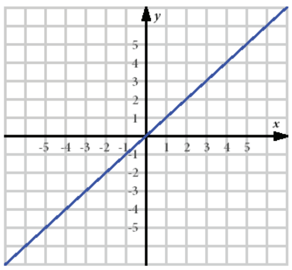
$$24 = 24$$

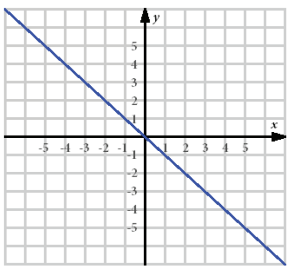
2 – Select Answer

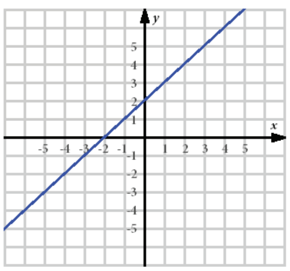
Select answer choice B.

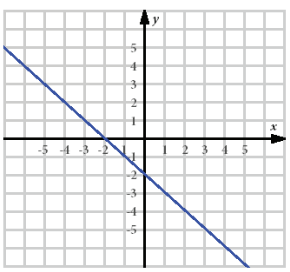
11

If k is a positive constant and different than 1, which of the following could be the graph of $y - x = k(y - x)$ in the xy-plane?

(A)


(B)


(C)


(D)


Solution

1 – Substitute Abstracts with Tangibles

The first step in this problem would be to remove the abstract and create a tangible. In this case, choose a positive number to replace k with other than 1. To start, we can make k = 2.

$$y - x = k(y - x)$$
$$y - x = 2(y - x)$$

Next, get the equation in slope-intercept form (y = mx + b)

$$y - x = 2y - 2x$$
$$y = x$$

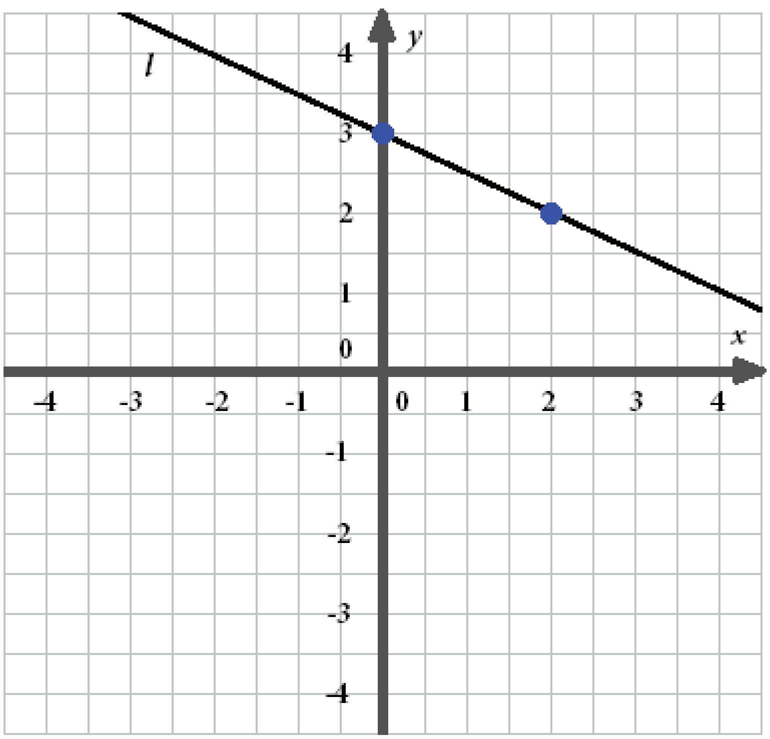
Now that this equation is in y = mx + b form, the slope of the graph needs to be positive 1 and a y-intercept of 0 (meaning that it goes through the origin).

2 – Select Answer

Select answer choice A.

12

Line l is graphed in the xy-plane below.



If line l is reflected around the origin, then what is the slope of the new line?

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

Solution

1 – Ace Graphs

Reflection about the origin would essentially be the same line just directly diagonal from the current line. Therefore, the slopes would remain the same.

Calculate the slope of the current line. Choose two points on the line: (0, 3) & (2, 2)

$$\frac{3-2}{0-2}$$

$$-\frac{1}{2}$$

2 – Select Answer

Select answer choice B.

13

The cost of a taxi from the airport to the city center can be represented using the equation $y = 3x + 5$, where x represents the number of miles driven. Which of the following statements is the best interpretation of the number 5 in this equation?

(A) The Initial Charge

(B) Rate Per Mile

(C) Rate Per Hour

(D) The Total Cost of the Trip

Solution

1 – Substitute Abstracts with Tangibles

Although some mathematically inclined students could quickly figure out what the 5 in the equation stands for, let's imagine that we have no idea. If we have no idea, let's try substituting the abstract x with a tangible number.

If x = 2 miles, how much would the taxi ride cost?

$$y = 3x + 5$$

$$y = 3(2) + 5$$

$$y = 6 + 5$$

$$y = 11$$

A taxi ride would cost $11 if we drove 2 miles. From the equation, we can see that it costs $6 just based on mileage. So why is it $11 for the total cost of the trip? The 5 must represent an extra charge.

2 – Select Answer

Select answer choice A.

14

If $\frac{t}{x+1}=\frac{t-1}{2}$ where x does not equal -1, what is x in terms of t ?

(A) $\frac{1+t}{1-t}$

(B) $\frac{1-t}{1+t}$

(C) $\frac{t+1}{t-1}$

(D) $\frac{t-1}{t+1}$

Solution

1 – Ace Equations

$$\frac{t}{x+1}=\frac{t-1}{2}$$

Start by cross multiplying to get rid of the fractions.

$$2t=(x+1)(t-1)$$

2 – Substitute Abstracts with Tangibles

Now let's say that t = 2. What would x be?

$$2(2)=(x+1)(2-1)$$

$$4=(x+1)$$

$$x=3$$

Now examine which answer choice gives us 3 when t = 2.

(A) $\frac{1+2}{1-2} = -3$

(B) $\frac{1-2}{1+2} = -\frac{1}{3}$

(C) $\frac{2+1}{2-1} = 3$

(D) $\frac{2-1}{2+1} = \frac{1}{3}$

3 – Select Answer

Select answer choice C.

15

If $\frac{x-1}{4}-\frac{y+1}{2}=1$, what is the value of $x-2y$?

(A) 1

(B) 2

(C) 4

(D) 7

Solution

1 – Ace Equations

$$\frac{x-1}{4}-\frac{y+1}{2}=1$$

To get rid of the fractions, start by multiplying everything by 4.

$$(x-1)-2(y+1)=4$$

$$(x-1)-(2y+2)=4$$

$$x+\ -1+\ -2y+\ -2=4$$

$$x+\ -2y+\ -3=4$$

$$x+\ -2y=7$$

*** Never get frustrated when the SAT gives you only one equation, but wants the value for 2 variables. Although you may think this is impossible, the SAT will often reward you with the answer with just a little manipulation of the equation.**

2 – Select Answer

Select answer choice D.

16

In the system of linear equations below, a is a constant. If the system has infinitely many solutions, what is the value of a ?

$$\begin{cases} \frac{2}{3}x + \frac{1}{2}y = 5 \\ 8x - ay = 60 \end{cases}$$

(A) -6

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 6

Solution

1 – Ace Graphs

Remember that identical lines have an infinite number of solutions. Therefore, we should think about how to get from one item in the first equation to a corresponding item in the second equation. How do we get from 5 to 60? Multiply by 12. Therefore, let's multiply the entire first equation by 12.

$$\left[\frac{2}{3}x + \frac{1}{2}y = 5\right]12$$

$$8x + 6y = 60$$

This looks identical to the first equation, except that 6 = –a. So a = –6.

2 – Select Answer

Select answer choice A.

17

Two workers together finish a job in 4 hours. One of them works 3 times faster than the other. The equation below represents the situation described.

$$\frac{1}{x}+\frac{3}{x}=\frac{1}{4}$$

Which of the following describes what the expression $\frac{3}{x}$ represents in the equation?

(A) The time, in hours, that it takes the slower worker to complete a job.

(B) The portion of the job that the slower worker would complete in three hours.

(C) The portion of the job that the faster worker would complete in one hour.

(D) The time, in hours, that it takes the faster worker to complete a the job.

Solution

1 – Ace Equations

Although some mathematically inclined students could quickly figure out what the $\frac{3}{x}$ in the equation stands for, let's imagine that we have no idea. If we have no idea, let's try solving for x.

$$\frac{1}{x}+\frac{3}{x}=\frac{1}{4}$$

$$\left[\frac{1}{x}+\frac{3}{x}=\frac{1}{4}\right]x$$

$$1+3=\frac{x}{4}$$

$$x=16$$

Now that we know x = 16, let's look at the original equation with the x-value plugged in.

$$\frac{1}{16} + \frac{3}{16} = \frac{1}{4}$$

Judging by the equation, it looks like $\frac{3}{16}$ represents the portion of the job that the faster worker completes. $\frac{1}{16}$ represents the portion of the job that the slower worker completes. And $\frac{1}{4}$ represents the portion of the job completed in 1 hour.

2 – Select Answer

Select answer choice C.

18

What is one possible solution to the equation below?

$$\frac{20}{x+1}-\frac{2}{x-2}=3$$

Solution

1 – Ace Equations

Start by finding a common denominator by multiplying each fraction by 1.

$$\frac{20}{x+1}\frac{x-2}{x-2}-\frac{2}{x-2}\frac{x+1}{x+1}=3$$

$$\frac{20(x-2)-2(x+1)}{(x+1)(x-2)}=3$$

üüxüℵℵℵℵ x $(x \quad)(x \quad)$

$$18x + \ -42 = 3(x^2 - x - 2)$$

$$18x + \ -42 = 3x^2 - 3x - 6$$

$$0 = 3x^2 \quad 21x \quad 36$$

$$0 = 3(x^2 - 7x + 12)$$

$$0 = 3(x-3)(x-4)$$

$ü = 3 \qquad 4$

2 – Fill in Answer

Fill in 3 or 4 as your answer.

19

A specific production line of a refinery produces only lubricants and sealants. The cost per barrel for the lubricants is $1,500 and for the sealants is $1,800. Which of the following inequalities represents the possible number of barrels of lubricants x and barels of sealants y the refinery could produce, in order not to exceed a total cost threshold of $25,000?

(A) $1{,}500x + 1{,}800y < 25{,}000$

(B) $1{,}500x + 1{,}800y \leq 25{,}000$

(C) $\frac{1{,}500}{x} + \frac{1{,}800}{y} \geq 25{,}000$

(D) $\frac{1{,}500}{x} + \frac{1{,}800}{y} > 25{,}000$

Solution

1 – Ace Inequalities

Interpret what "not to exceed" means. This means that the cost of lubricants and sealants has to be less than or equal to $25,000.

2 – Select Answer

Select answer choice B.

20

A researcher is interested in estimating the average time a mother with one child up to 4 years old spends watching TV between 5pm to 10pm. She surveyed 80 mothers of the above population and found the sample mean to be 34 minutes while the margin of error for this estimate was 3.1 minutes. If she were to replicate the sampling in an attempt to get a smaller margin of error, which of the following samples will most likely result in a reduced margin of error for the mean time spent watching TV between 5pm and 10pm by a mother with one child up to 4 years old?

(A) 50 randomly selected mothers with one child up to four years old

(B) 170 randomly selected mothers with multiple children up to four years old

(C) 250 randomly selected mothers with multiple children up to four years old

(D) 320 randomly selected mothers with one child up to four years old

Solution

1 – Ace Center of Data

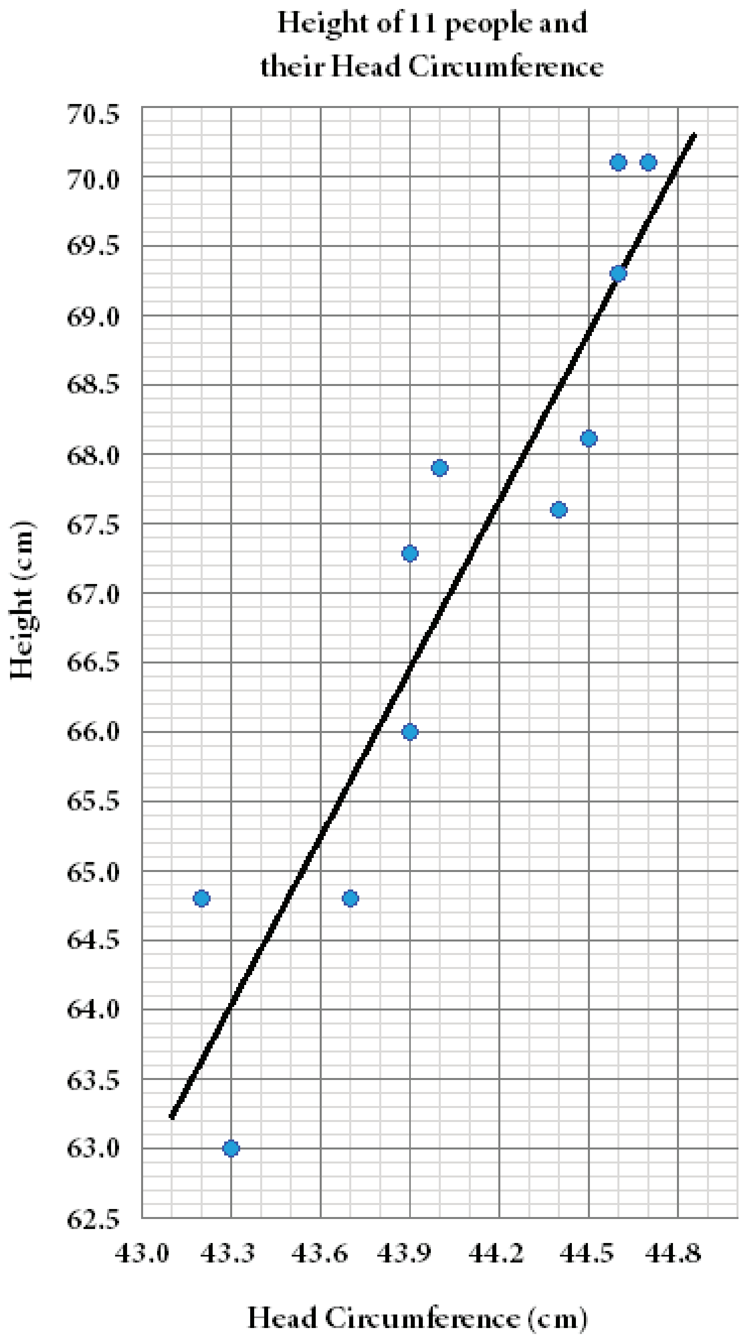
In order to decrease the margin of error, you must increase the sample size of the same population.

2 – Select Answer

Select answer choice D.

21

The scatterplot below shows the relationship between the head circumference and height for 11 people.



How many of the eleven people have an actual height that differs at least one centimeter from the height predicted by the line of best fit?

(A) 0

(B) 1

(C) 3

(D) 9

Solution

1 – Ace Data Analysis

Whenever you're given a line graph, you should interpret one point. The first point says that a person with a head circumference of 43.2cm has a height of 64.8cm.

Now determine how many points on the graph differ from the line of best fit by at least 1cm.

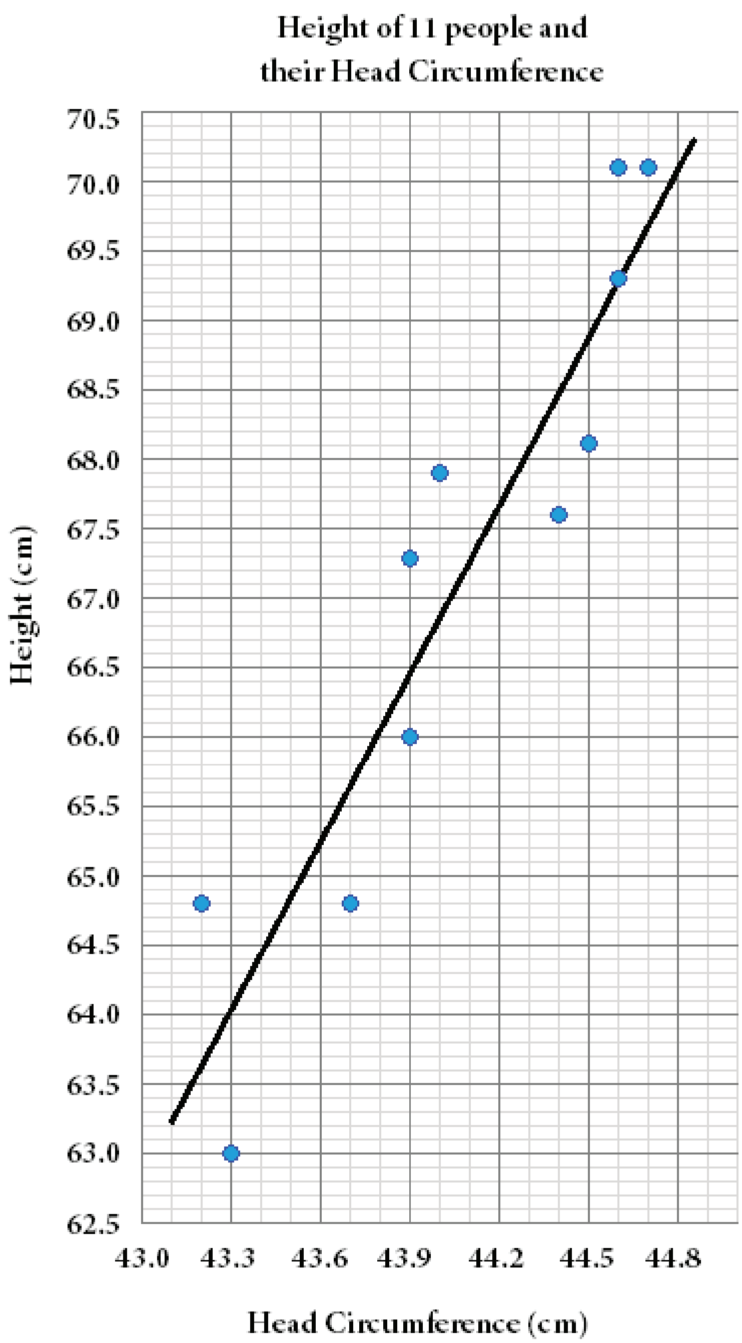
- → **The point at 43.2cm**
- → **The point at 43.3cm**
- → **The point at 44.0cm**

2 – Select Answer

Select answer choice C.

22

The scatterplot below shows the relationship between the head circumference and height for 11 people.



Which of the following is the best interpretation of the slope of the line of best fit in the context of this problem?

(A) The predicted head circumference increase in centimeters for every centimeter increase in the height
(B) The predicted height increase in centimeters for one inch increase in the head circumference
(C) The predicted height in centimeters of a person with at least 43.1 centimeters head circumference
(D) The predicted height increase in centimeters for one centimeter increase in the head circumference

Solution

1 – Ace Data Analysis

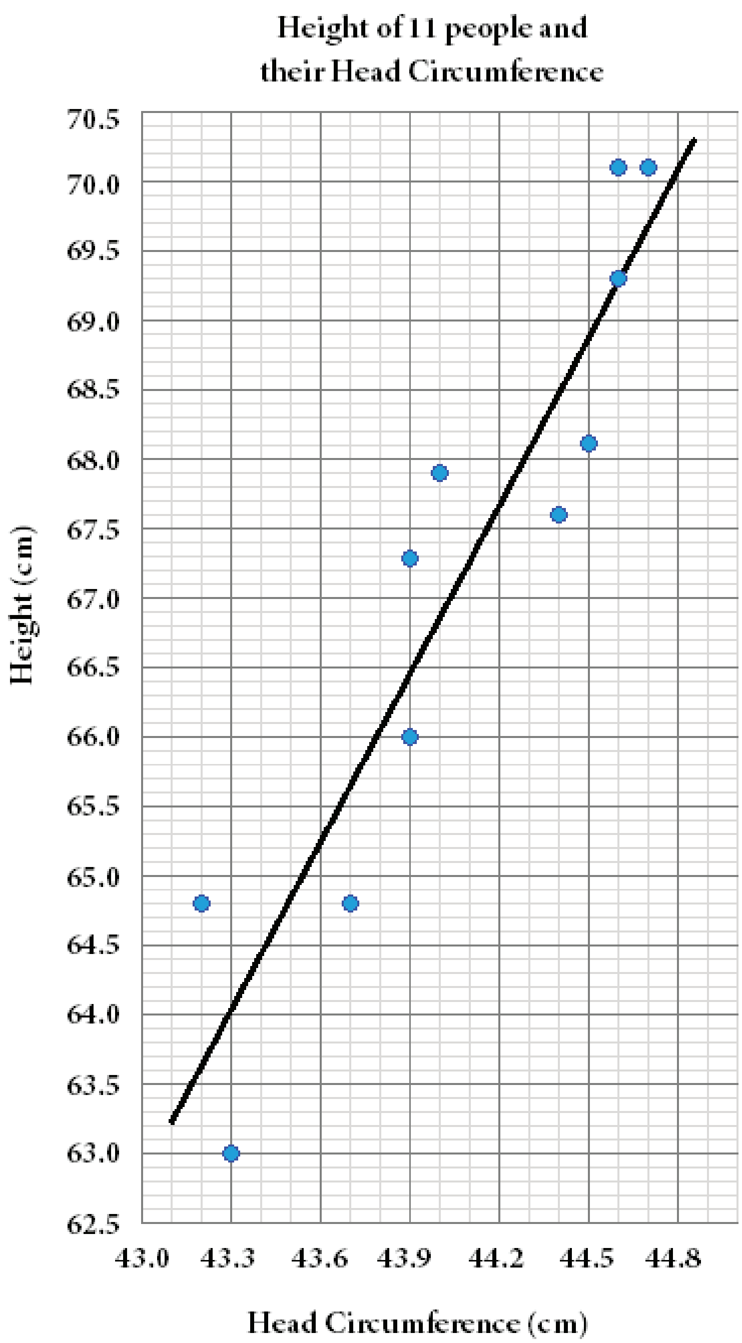
A line of best fit attempts to predict the change in y with respect to the change in x. Therefore, the line of best fit attempts to predict the change in height that will occur with a change in head circumference.

2 – Select Answer

Select answer choice D.

23

The scatterplot below shows the relationship between the head circumference and height for 11 people.



Based on the line of best fit, what is the best approximation of predicted height for someone with head circumference of 44.45 centimeters?

(A) 67.5 centimeters
(B) 68.7 centimeters
(C) 69.5 centimeters
(D) 69.9 centimeters

Solution

1 – Ace Data Analysis

Find 44.45 centimeters on the x-axis.

Find the what y-value the line of best fit is at when x = 44.45: approximately 68.7cm

2 – Select Answer

Select answer choice B.

24

If $\frac{4}{3} < 5a - 2 < \frac{7}{4}$, what is one possible value of 3 + 10*a* ?

Solution

1 – Ace Inequalities

$$\frac{4}{3} < 5a - 2 < \frac{7}{5}$$

$$\frac{10}{3} < 5a < \frac{17}{5}$$

$$\frac{50}{15} < 5a < \frac{51}{15}$$

$$\frac{50}{75} < 5a < \frac{51}{75}$$

$$\frac{100}{150} < a < \frac{102}{150}$$

One possible value of a is 101/50.

$$3 + 10a$$

$$3 + 10\left(\frac{101}{50}\right)$$

$$3 + \frac{1010}{50}$$

$$3 + 20.20$$

$$23.2$$

2 – Fill in Answer

Fill in 23.2 as the answer.

25

A public opinion survey was conducted in order to explore the relationship between age and support for reforming the state health care system. The table below displays a summary of the survey results.

Reported Opinion on Health Care System Reform by Age				
	For	Against	No Opinion	Total
18- to 34- years old	182	251	198	631
35- to 54- years old	214	283	115	612
55- to 74- years old	453	162	81	696
People 75 years old and over	527	121	58	706
Total	1,376	817	452	2,645

According to the table, which age group did the greatest percentage of people have the opinion that they are against the reform?

(A) 18- to 34- year olds

(B) 35- to 54- year olds

(C) 55- to 74- year olds

(D) People 75 years old and over

Solution

1 – Ace Data Analysis

Start by reading one line of the table.

Of 631 people age 18 to 34-years old, 182 supported reforming the state health care system, 251 did not support reforming the state health care system, and 198 had no opinion.

To determine which age group had the greatest percentage of people against the reform, look at the number of people against the reform over the total number of people in each age group.

18-34 year olds: $\frac{251}{631}$

35-54 year olds: $\frac{283}{612}$

55-74 year olds: $\frac{162}{696}$

75+ : $\frac{121}{706}$

Without even having to calculate the actual percentages, we can tell that 35-54 year olds will have the highest percentage (highest numerator and smallest denominator).

2 – Select Answer

Select answer choice B.

26

A public opinion survey was conducted in order to explore the relationship between age and support for reforming the state health care system. The table below displays a summary of the survey results.

Of the 55- to 74-year-olds who support the reform of the state health care system, 120 people were randomly selected for a follow-up survey where they were asked whether they are covered by a private health insurance policy. 48 people in this follow-up survey sample said they are covered by a private health insurance policy while the rest are not.

Reported Opinion on Health Care System Reform by Age				
	For	Against	No Opinion	Total
18- to 34- years old	182	251	198	631
35- to 54- years old	214	283	115	612
55- to 74- years old	453	162	81	696
People 75 years old and over	527	121	58	706
Total	1,376	817	452	2,645

Using the data from both the follow-up survey and the initial survey, which of the following is most likely to be an accurate statement?

(A) About 181 people 55 to 74 years old are covered by private health insurance policy

(B) About 277 people 55 to 74 years old who support the state health care system reform are covered by private health insurance policy

(C) About 271 people 55 to 74 years old who support the state health care system reform are not covered by private health insurance policy

(D) About 40% of people 55 to 74 years old who support the state health care system reform are covered by private health insurance policy

Solution

1 – Ace Data Analysis

Calculate the percentage of health care reform supporters 55 to 74-years old who are covered by private health insurance using information from the follow-up survey.

$$\frac{48}{120}$$

$$40\%$$

2 – Select Answer

Select answer choice D.

27

A researcher wanted to know if there is an association between watching TV and sleeping problems for the population of 15-year-olds in Germany. He obtained survey responses from a random sample of 2200 15-year-old Germans and found convincing evidence of a positive association between watching TV and sleeping problems.

Which of the following conclusions is well supported by the data?

(A) There is a positive association between watching TV and sleeping problems for 15-year-olds in Europe.

(B) Using watching TV and sleeping problems as defined by the study, an increase in sleeping problems is caused by an increase in watching TV for 15-year-olds in Germany.

(C) Using watching TV and sleeping problems as defined by the study, an increase in sleeping problems is caused by an increase in watching TV for 15-year-olds in Europe.

(D) There is a positive association between watching TV and sleeping problems for 15-year-olds in Germany.

Solution

1 – Ace Data Analysis

Remember the difference between correlation and causation. Correlation implies association. Causation implies one item produces the other.

Before looking at the answer choices, try to determine what the data statement means in your own words: For 15-year old Germans, there is a positive correlation between watching TV and sleeping problems.

2 – Select Answer

Select answer choice D.

28

The ink cartridge of a photocopy machine has a capacity of 78 standard pages per milliliter of ink when the machine operates at the standard mode of 40 pages per minute. The ink cartridge has 19 milliliters of ink at the beginning of the printing process. If the photocopy machine operates at the standard mode of 40 pages per minute, which of the following functions *f* models the number of milliliter of ink remaining in the cartridge t minutes after the printing process begins?

(A) $f(t) = 19 - \frac{78}{40t}$

(B) $f(t) = \frac{19 - 78t}{40}$

(C) $f(t) = 19 - \frac{40t}{78}$

(D) $f(t) = \frac{19 - 40t}{78}$

Solution

1 – Ace Data Analysis

Let's substitute 2 minutes for t.

If the photocopy machine runs for 2 minutes, then 80 pages have been printed (40 pages per minute in standard mode x 2 minutes).

Calculate the number of milliliters of ink that have been used to print 80 pages.

$$80 \text{ pages} \times \frac{1 \text{ mL}}{78 \text{ pages}}$$

$$1.03 \text{ mL}$$

Calculate the number of milliliters remaining.

$$19\ mL - 1.03\ mL$$

$$17.97\ mL$$

Which answer choice results in 17.97 mL when you plug in 2 minutes?

(A) $f(t) = 19 - \dfrac{78}{40(2)}$

(B) $f(t) = \dfrac{18 - 78(2)}{40}$

(C) $f(t) = 19 - \dfrac{40(2)}{78}$

(D) $f(t) = \dfrac{19 - 40(2)}{78}$

You should be able to quickly tell without a calculator that B and D are not the answer because they would result in negative numbers.

2 – Select Answer

Select answer choice C.

29

A convenience store sells small bottles of orange juice for \$2.35 each and large bottles of orange juice for \$2.95 each. During a three-hour period, a total of 45 small and large bottles of orange juice have been sold and the total amount collected from these was \$115.95. Solving which of the following systems of equations yields the number of small bottles of orange juice, x, and the number of large bottles of orange juice, y, that were sold during the three hours?

(A) $\begin{cases} x+y=45 \\ 2.95x+2.35y=115.95 \end{cases}$

(B) $\begin{cases} x+y=45 \\ 2.35x+2.95y=\dfrac{115.95}{3} \end{cases}$

(C) $\begin{cases} x+y=45 \\ 2.35x+2.95y=115.95 \end{cases}$

(D) $\begin{cases} x+y=\dfrac{45}{3} \\ 2.35x+2.95y=\dfrac{115.95}{3} \end{cases}$

Solution

1 – Ace Equations

Small bottles of orange juice are represented by x and cost \$2.35

\$2.35 will be the coefficient on x

Large bottles of orange juice are represented by y and cost $2.95

$2.95 will be the coefficient on y

The total price of orange juice sold was $115.95

$$\$2.35x + \$2.95y = \$115.95$$

The total number of small bottles and large bottles of orange juice sold was 45

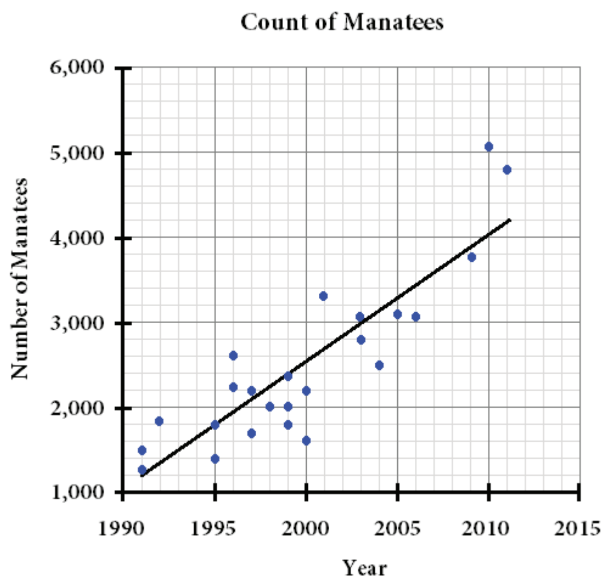
$$x + y = 45$$

2 – Select Answer

Select answer choice C.

30

The scatterplot below shows counts of Florida manatees, a type of sea mammal, from 1991 to 2011. If t is an integer representing the year, where $1990 < t < 2012$ and y represents the number of manatees, which of the following equations is the equation of the line of best fit to the data that is shown in the scatterplot?



(A) $y = 75t + 1100$

(B) $y = 150t + 1200$

(C) $y = 150\,(t - 1990) + 2000$

(D) $y = 150\,(t - 1990) + 1050$

Solution

1 – Ace Graphs

Simply plug in a number. I will randomly choose 2000.

Find what y-value on the line of best fit corresponds with 2000 on the x-axis.

Approximately 2500 manatees.

Determine which of the equations in the answer choices results in approximately 2500 manatees when you plug in 2000 for t.

(A) $y = 75\,(2000) + 1100$

(B) $y = 150\,(2000) + 1200$

(C) $y = 150\,(2000 - 1990) + 2000$

(D) $y = 150\,(2000 - 1990) + 1050$

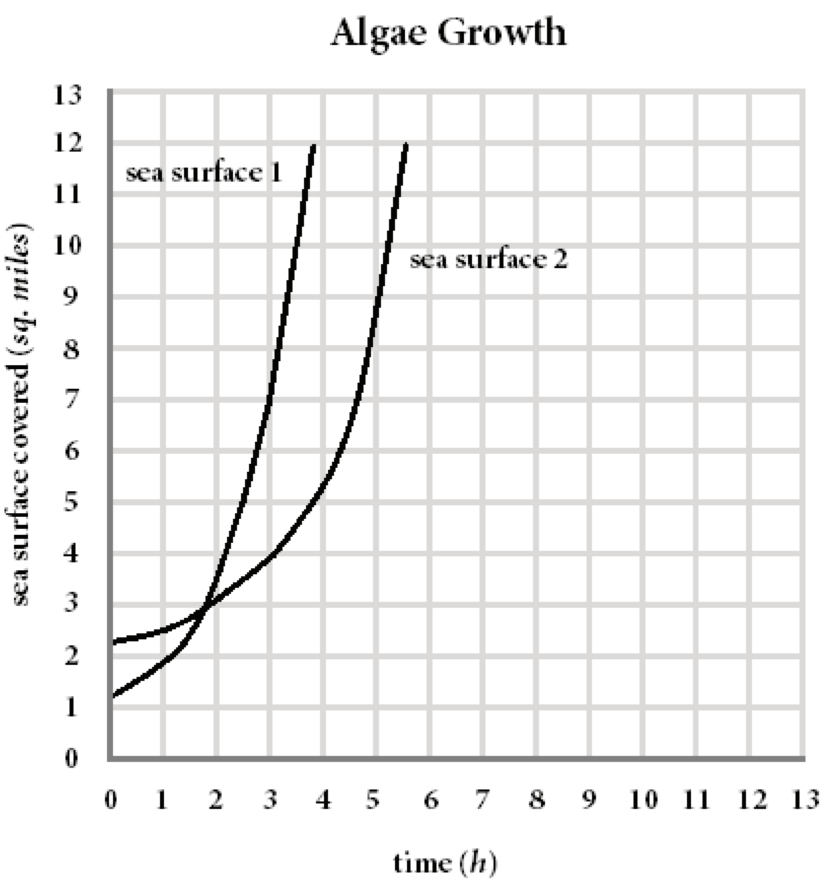
You should not need a calculator to be able to quickly eliminate answer choices A, B, and C.

2 – Select Answer

Select answer choice D.

31

In some parts of the world, the red tide is an annual event. During the red tide, the sea changes color as algae take over the surface with exponential growth. Scientists select two different sea areas of 12 sq. miles each and record the sea surface covered by algae every 15 minutes. The data collected for each sea area were fit by a smooth curve, as shown, where each curve represents the surface of the sea covered by algae as a function of time in hours. Which of the following is a correct statement about the data?



(A) At time $t = 0$, the sea surface 2 is covered by twice as much algae as sea surface 1.

(B) For the first hour, the area covered in sea surface 2 is increasing at a higher average rate than the area covered in sea surface 1.

(C) The sea surface 1 becomes fully covered earlier than sea surface 2 by more than 1 hour.

(D) About 15 minutes after the second hour begins, the same percentage of both sea surfaces is covered by algae.

Solution

1 – Ace Graphs

Before looking at the answer choices, we should make some interpretations about the graph. Let's analyze each of the end points of each line.

Sea Surface 1 – The entire 12-square mile surface is covered in about 3.8 hours

Sea Surface 2 – The entire 12-square mile surface is covered in about 5.4 hours

We can conclude that the algae on sea surface 1 moves faster.

Let's analyze each of the answer choices

(A) Sea surface 1 seems to have about 1.2 square miles covered at t = 0. Sea surface 2 seems to have about 2.1 square miles covered at t=0. 2.1 is not double 1.2

(B) For the first hour, sea surface 1 actually seems to have a steeper positive slope than sea surface 2.

(C) Yes! This is true based on our original analysis of the graph.

(D) No. The same percentage of both sea surfaces is covered about 15 minutes before the

second hour begins.

2 – Select Answer

Select answer choice C.

32

If (x, y) is a solution to the system of equations below and y does not equal 0, what is the value of $(x+y)^3$?

$$\begin{cases} x^2 - y^2 = y \\ x = -3y \end{cases}$$

(A) $\frac{1}{8}$

(B) $\frac{1}{64}$

(C) $\frac{1}{16}$

(D) $-\frac{1}{64}$

Solution

1 – Ace Equations

Start by substituting x = – 3y into the top equation.

$$(-3y)^2 - y^2 = y$$

$$9y^2 - y^2 = y$$

$$8y^2 = y$$

$$8y^2 - y = 0$$

$$y(8y - 1) = 0$$

y could equal 0, but the question specifically states that this is not the case. Therefore, let's see what y would be if 8y – 1 = 0.

$$8y - 1 = 0$$

$$8y = 1$$

$$y = \frac{1}{8}$$

Now find the value of x.

$$x = -3y$$

$$x = -3\left(\frac{1}{8}\right)$$

$$x = -\frac{3}{8}$$

Now find the value of the unknown.

$$(x + y)^3$$

$$\left(-\frac{3}{8} + \frac{1}{8}\right)^3$$

$$\left(-\frac{2}{8}\right)^3$$

$$-\frac{8}{512}$$

$$-\frac{1}{64}$$

2 – Select Answer

Select answer choice D.

33

An international bank issues its Traveler credit cards worldwide. When a customer makes a purchase using a Traveler card in a currency different from the customer's home currency, the bank converts the purchase price at the daily foreign exchange rate and then charges a 2% fee on the converted cost.

Peter lives in the United States, but is on vacation in Thailand. He used his Traveler card for a purchase that cost 580 Baht (Thailand currency). The bank posted a charge of $18.10 to his account that included the 2% fee.

What foreign exchange rate, in U.S. dollar per one Baht, did the bank use for Peter's charge? Round your answer using three decimal places.

Solution

1 – Ace Ratios & Percentages

Let's use our knowledge of included percentages in order to solve this question. We know that 580 Baht resulted in $18.10 after a 2% fee was charged. So let's start by calculating how much Peter would have gotten without the 2% tax.

$$1.02x = \$18.10$$
$$x = \$17.75$$

If there was no 2% tax, then Peter would have received $17.75. Given this information, let's calculate many dollars are in 1 Baht.

$$\frac{\$17.75}{580 \text{ Baht}}$$

$$.031 \text{ dollars per Baht}$$

The exchange rate in dollars per Baht is .031.

2 – Fill in Answer

Fill in ".031" into the free response grid-in answer sheet.

34

The function *f* is defined as $f(x) = 2x^3 + ax^2 + bx - 48$. In the *xy*-plane, the graph of *f* intersects the *x*-axis at three points: (–4, 0), (–2, 0), and (3, 0). What are the values of *a* and *b* ?

(A) $a = 6,\ b = -20$

(B) $a = \frac{14}{5},\ b = -\frac{52}{5}$

(C) $a = \frac{14}{5},\ b = \frac{52}{5}$

(D) $a = \frac{34}{5},\ b = -\frac{92}{5}$

Solution

1 – Substitute Answers in Problem

Remember that coordinates are solutions. Therefore, plug in one of the given solutions into the equation.

$$f(x) = 2x^3 + ax^2 + bx - 48$$

$$0 = 2(-4)^3 + a(-4)^2 + b(-4) - 48$$

$$0 = 2(-64) + a(16) + b(-4) - 48$$

$$48 = (-128) + a(16) + b(-4)$$

$$176 \aleph 16a \qquad 4b$$

Now just plug in the answer choices to see which one makes the equation true.

(A) $176 = 16(6) - 4(-20)$

(B) $176 = 16\left(\frac{14}{5}\right) - 4\left(-\frac{52}{5}\right)$

(C) $176 = 16\left(\frac{14}{5}\right) - 4\left(\frac{52}{5}\right)$

(D) $176 = 16\left(\frac{34}{5}\right) - 4\left(-\frac{92}{5}\right)$

2 – Select Answer

Select answer choice A.

35

The mean height y in centimeters for x months old children is estimated using the equation $y = .635x + 64.928$, where $18 < x < 29$. Which of the following statements is the best interpretation of the numbers 0.635 and/or 64.928 in the context of this problem?

(A) 64.928 cm is the minimum estimated mean height for children between 18 and 29 months old.

(B) 64.928 cm is the estimated mean height for children 18 months old.

(C) 0.635 cm is the estimated increase in the mean height of children per monthly increase of their age.

(D) 0.635 + 64.928 cm is the estimated mean height for 18 months old children.

Solution

1 – Substitute Abstract with Tangibles

Although some mathematically inclined students could quickly figure out what the 0.635 and 64.928 in the equation stand for, let's imagine that we have no idea. If we have no idea, let's try plugging in a number for x and seeing what happens.

If x = 20 months, what is the predicted height of the child?

$$y = .635x + 64.928$$

$$y = .635(20) + 64.928$$

$$y = 77.628$$

A child's height would be predicted to be 77.628cm if the child is 20 months old. From the equation, we can see that the 0.635 is likely the additional height the child gets each month.

2 – Select Answer **Select answer choice C.**

36

If $\frac{3a}{a-2}=\frac{6}{5y}$ where y does not equal 0 and a does not equal 2, what is y in terms of a ?

(A) $=\frac{6 \quad -2}{15}$

(B) $y=\frac{2}{5}-\frac{4}{5a}$

(C) $y=\frac{2a}{5}-\frac{4}{5}$

(D) $y=-\frac{12}{15a-6}$

Solution

1 – Substitute Abstract with Tangibles

Let's start by plugging in any value for a. I will randomly select 3.

$$\frac{3a}{a-2}=\frac{6}{5y}$$

$$\frac{3(3)}{\text{ü}-}=\frac{6}{y}$$

$$9=\frac{6}{5y}$$

$$45y=6$$

$$y=\frac{6}{45}=\frac{2}{15}$$

Which answer choice works out when we plug in a = 3 or $y = \frac{2}{15}$?

(A) $y = \frac{ü(\ \ )-}{ü\ (\ \ )}$

(B) $y = \frac{2}{ü} - \frac{4}{(\ \ )}$

(C) $y = \frac{2(3)}{5} - \frac{4}{5}$

(D) $y = -\frac{12}{15(3)-6}$

Note that you can certainly solve this question using traditional high school algebra. I simply wanted to show you how you could remove the algebra from the problem using Substitute Abstract with Tangibles.

2 – Select Answer

Select answer choice B.

37

If $m = x^2 \quad 3x \quad 2$ and $n = x^3 + 3x - 1$, what is $m^2 - 2n$ in terms of x?

(A) $x^4 - 8x^3 + 13x^2 - 18x + 6$

(B) $x^4 - 2x^3 - 9x^2 - 6x + 6$

(C) $x^4 - 8x^3 + 13x^2 - 15x + 5$

(D) $x^4 - 2x^3 - 5x^2 - 18x + 6$

Solution

1 – Ace Expressions

Simply work carefully and methodically.

$$m^2 - 2n$$

$$\left(x^2 - 3x + 2\right)^2 - 2\left(x^3 + 3x - 1\right)$$

$$\left(x^2 - 3x + 2\right)\left(x^2 - 3x + 2\right) - 2\left(x^3 + 3x - 1\right)$$

$$\left(x^4 - 3x^3 + 2x^2 - 3x^3 + 9x^2 - 6x + 2x^2 - 6x + 4\right) - 2\left(x^3 + 3x - 1\right)$$

*** When there are so many signs, it's easy to make a mistake. So I like to convert all subtractions to the addition of a negative.**

$$\left(x^4 + -6x^3 + 13x^2 + -12x + 4\right) + \left(-2x^3 + -6x + 2\right)$$

$$x^{\text{ü}} + -8x \;\; + 13x \;\; + -18x + 6$$

2 – Select Answer

Select answer choice A.

38

$$\frac{3(x^2-5)-2x}{x+1}=\frac{4-(5-6x)}{2}-2$$

In the equation above, what is the value of x ?

(A) -5

(B) $-\frac{27}{7}$

(C) -1

(D) -7

Solution

1 – Substitute Answers in Problem

Substituting the answers back into the original equation can work. However, if this problem is on the no-calculator portion of the SAT section, then plugging in the answer choices would be difficult. Nevertheless, we know that answer choice C is incorrect because –1 would result in 0 in the denominator of the fraction on the left side of the equation.

2 – Ace Equations

$$\frac{3\left(x^2-5\right)-2x}{x+1}=\frac{4-\left(5-6x\right)}{2}-2$$

$$\frac{3\left(x^2-5\right)-2x}{x+1}=\frac{4-\left(5-6x\right)}{2}-\frac{4}{2}$$

$$\frac{3\left(x^2-5\right)-2x}{x+1}=\frac{4-\left(5-6x\right)-4}{2}$$

$$\frac{3x^2+\ -15+\ -2x}{x+1}=\frac{4+\ -5+6x+\ -4}{2}$$

$$\frac{3x^2+\ -2x+\ -15}{x+1}=\frac{-5+6x}{2}$$

$$2\left(3x^2+\ -2x+\ -15\right)=\left(6x-5\right)\left(x+1\right)$$

$$6x^2+\ -4x+\ -30=6x^2+6x-5x-5$$

$$6x^2+\ -4x+\ -30=6x^2+x-5$$

$$-4x+\ -30=x-5$$

$$-25=5x$$

$$-5=x$$

3 – Select Answer

Select answer choice A.

39

If $\frac{3}{4}x+\frac{9}{5}y=1$, what is the value of $5x + 12y$?

Solution

1 – Ace Equations

Remember that if the SAT gives you one equation, but wants the value of two variables – you can often manipulate the equation so that you can find the value of the unknown expression.

Start by figuring out how to get from $\frac{3}{4}x$ to 5x.

$$\frac{5}{\frac{3}{4}}$$

$$5\cdot\frac{4}{3}$$

$$\frac{20}{3}$$

$\frac{3}{4}$ must be multiplied by $\frac{20}{3}$ in order to get 5.

Let's try multiplying the entire equation by

$$\frac{20}{3}$$

$$\frac{3}{4}x + \frac{9}{5}y = 1$$

$$\left[\frac{3}{4}x + \frac{9}{5}y = 1\right]\frac{20}{3}$$

$$\frac{60}{12}x + \frac{180}{15}y = \frac{20}{3}$$

$$5x + 12y = \frac{20}{3}$$

2 – Fill in Answer

Fill in 20/3

40

Which of the following is equal to $\sin(\pi + x)$ where $0 < x < \frac{\pi}{6}$?

(A) $\sin(x)$

(B) $\sin(\pi - x)$

(C) $\cos\left(\frac{3\pi}{2} - x\right)$

(D) $\cos\left(\frac{\pi}{2} + x\right)$

Solution

1 – Ace Trigonometric Equations

Start by choosing an angle for x.

$$x = \frac{\pi}{12}$$
$$x = 15°$$

Which angle would be equivalent to sin(180 + 15°) or sin(195°) $\frac{\pi}{2}$

Realize that sin(195°) is in the third quadrant. Therefore, the only other sine that could be equivalent to it would be in the fourth quadrant (since the y is negative). However, sin(15°) and sin(75°) are not in the third or fourth quadrant. Therefore, A and B are not correct.

Remember that when converting from sin to cos, you must subtract the angle from

$$\sin(195°) = \cos(90° - 195°)$$

$$\sin(195°) = \cos(-105°)$$

Realize that cos (–105°) = cos (360° – 105°) = cos (255°)

Examine answer choices C and D

$$\text{(C) } \cos(270° - 15°) = \cos(255°)$$

$$\text{(D) } \cos(90° + 15°) = \cos(105°)$$

2 – Select Answer

Select answer choice C.

41

$$\begin{cases} \frac{1}{3}x - \frac{3}{4}y = 5 \\ ax - 6y = b \end{cases}$$

In the system of linear equations above, a and b are constants. If the system has infinite solutions, what are the values of a and b ?

(A) $a = \frac{3}{2},\ b = \frac{5}{2}$

(B) $a = \frac{3}{2},\ b = 5$

(C) $a = \frac{8}{3},\ b = 5$

(D) $a = \frac{8}{3},\ b = 40$

Solution

1 – Ace Graphs

Remember that identical lines have an infinite number of solutions. Therefore, we should think about how to get from one item in the first equation to a corresponding item in the second
equation. How do we get from $-\frac{3}{4}$ to –6? Multiply by 8. Therefore, let's multiply the entire first equation by 8.

This looks identical to the second equation.

2 – Select Answer

Select answer choice D.

42

$$19-\frac{60t}{24}$$

The gas mileage for a car is 24 miles per gallon when the car travels at an average speed of 60 miles per hour. The car's gas tank has 19 gallons of gas at the beginning of a trip. The expression above models the number of gallons of gas remaining in the tank t hours after the trip begins, provided that the car travels at an average speed of 60 miles per hour. Which of the following describes what the ratio $\frac{60}{24}$ represents in this expression?

(A) The miles the car has traveled after t hours at an average speed of 60 miles per hour

(B) The gallons per mile the car is consuming for 1 hour of trip at an average speed of 60 miles per hour

(C) The gallons per hour the car is consuming at an average speed of 60 miles per hour

(D) The time it takes for 1 gallon of gas to be consumed at an average speed of 60 miles per hour

Solution

1 – Substitute Abstract with Tangibles

Although some mathematically inclined students could quickly figure out what the in the equation stands for, let's imagine that we have no idea. If we have no idea, let's try plugging in a number for t and seeing what happens. $\frac{60}{24}$

If t = 2 hours, how many gallons would be remaining in the tank?

$$19 - \frac{60(2)}{24} = 19 - 5 = 14 \text{ gallons}$$

After substituting abstract with tangibles, it becomes clear that the $\frac{60}{24}$ **represents the rate of gas usage based on hours travelled at 60 miles per hour.**

2 – Select Answer

Select answer choice C.

43

The table below classifies the responses given by 68 adult men and women about their favorite leisure activity among Reading, Sports and TV.

	Reading	Sports	TV	Total
Men	9	15	11	35
Women	12	8	13	33
Total	21	23	24	68

What fraction of all men that don't prefer to watch TV, choose Sports as their favorite activity?

Solution

1 – Ace Data Analysis

9 men prefer to read and 15 men prefer sports, which represents 24 total men who do not prefer to watch TV.

$$\frac{15}{24} = \frac{5}{8}$$

2 – Fill in Answer

Fill in 5/8

44

The table below classifies the responses given by 68 adult men and women about their favorite leisure activity among Reading, Sports and TV.

	Reading	Sports	TV	Total
Men	9	15	11	35
Women	12	8	13	33
Total	21	23	24	68

What fraction of all women who prefer Reading or Sports, choose Sports as their favorite activity?

Solution

1 – Ace Data Analysis

12 women prefer reading and 8 women prefer sports, which represents 20 total women who prefer either reading or sports.

$$\frac{8}{20} = \frac{2}{5}$$

2 – Fill in Answer

Fill in 2/5

45

The students of a class took a biology test. The male students had an average score of 72, while the female students had an average score of 78. If the number of male students is greater than the number of female students, which of the following must be true about the mean score m of the combined group of students in the class?

(A) $m = 75$

(B) $m > 75$

(C) $72 < m \leq 74$

(D) $72 < m < 78$

Solution

1 – Ace Data Analysis

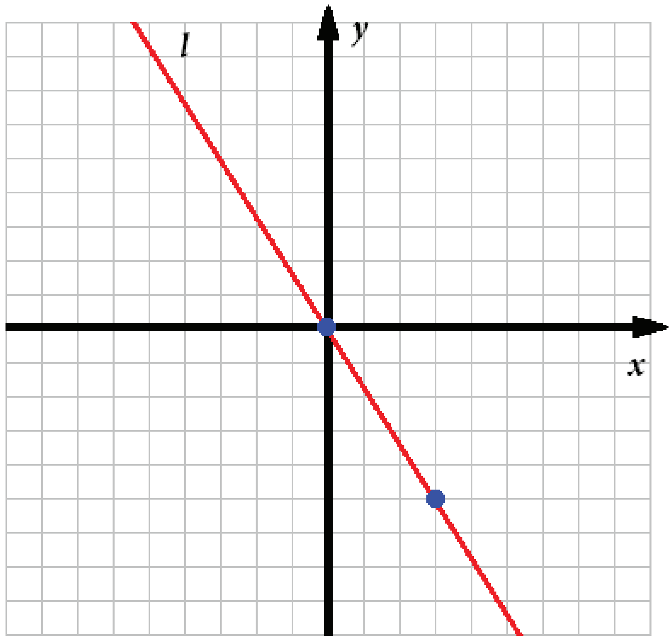
The total mean must be between the means of the two separate groups that make up the total.

2 – Select Answer

Select answer choice D.

46

Line l is graphed in the xy-plane below.



If line l is translated up 2 units and to the left 10 units, then what is the slope of the new line?

(A) $\frac{5}{3}$

(B) $\frac{5}{7}$

(C) $\frac{3}{2}+2$

(D) $-\frac{5}{3}$

Solution

1 – Ace Graphs

Select two points on the current line and find the slope: (0, 0) & (3, –5)

$$\frac{-5-0}{3-0}$$

$$-\frac{5}{3}$$

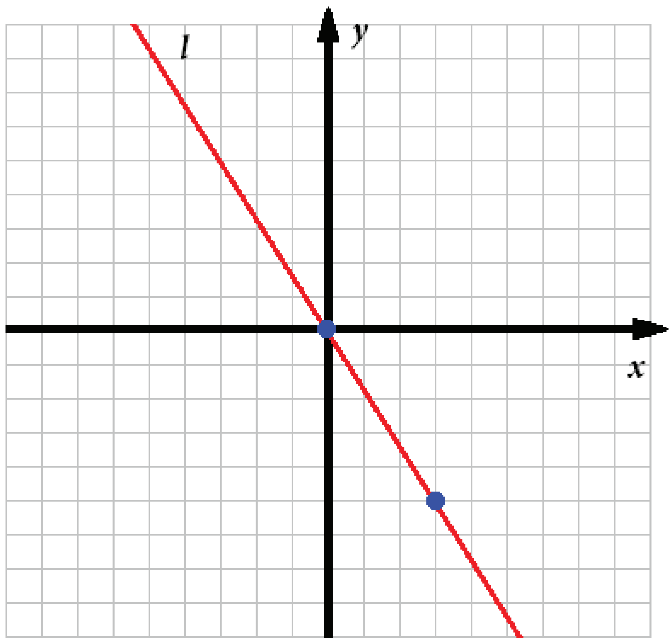
Translations do not change the slope of a line.

2 – Select Answer

Select answer choice D.

47

Line l is graphed in the xy-plane below.



What is the slope of a line perpendicular to line l ?

(A) $-\frac{3}{5}$

(B) $\frac{7}{5}$

(C) $\frac{2}{3}+2$

(D) $\frac{3}{5}$

Solution

1 – Ace Graphs

From the previous question, we know that the slope of the line is $-\frac{5}{3}$.

The opposite reciprocal $-\frac{5}{3}$ of $\frac{3}{5}$ is .

2 – Select Answer

Select answer choice D.

48

$$\begin{cases} 3(x+4)-2y=7(x-y) \\ \frac{1}{2}(x-1)+2y=3(x+y)-\frac{1}{2} \end{cases}$$

Based on the system of equations above, what is the value of the ratio of ?

(A) $-\frac{60}{29}$

(B) $\frac{8}{20}$

(C) $-\frac{2}{5}$

(D) $\frac{60}{29}$

Solution

1 – Ace Equations

Although this system of equations looks intimidating, Just Get Started. Start by simplifying just one equation.

$$3(x+4)-2y=7(x-y)$$

$$3x+12+\ -2y=7x+\ 7y$$

$$12+5y=4x$$

Now simplify the other equation.

$$\frac{1}{2}(x-1)+2y=3(x+y)-\frac{1}{2}$$

$$2\left[\frac{1}{2}(x-1)+2y=3(x+y)-\frac{1}{2}\right]$$

$$(x-1)+4y=6(x+y)-1$$

$$x+\ -1+4y=6x+6y+\ -1$$

$$-1=5x+2y+\ -1$$

$$0=5x+2y$$

$$-2y=5x$$

$$-\frac{2}{5}=\frac{x}{y}$$

Simplifying the second equation got us what we needed without even needing to work with the first equation!

2 – Select Answer

Select answer choice C.

Top 10 Most Common Hard SAT Math Questions

Combining Ace Exponents & Ace Expressions

1

If $\frac{x^{a^4}}{x^{b^4}} \quad x^{32}$, $x > 1$ and $a^2 - b^2 = 4$, what is the value of $a^2 + b^2$?

(A) 4

(B) 8

(C) 16

(D) 32

Solution

1 – Ace Exponents & Ace Expressions

Division of exponents on common bases should be subtracted.

$$x^{a^4 - b^4} = x^{32}$$

Because the bases are the same, you can simply set the exponents equal to each other.

$$a^4 - b^4 = 32$$

Factor a perfect square expression.

$$\left(a^2+b^2\right)\left(a^2-b^2\right)=32$$

$$\left(a^2+b^2\right)4=32$$

$$\left(a^2+b^2\right)=8$$

2 – Select Answer

Select answer choice B.

Rationalizing Denominators With Complex Numbers

2

$$\frac{4-i}{5-3i}$$

If the expression above is rewritten in the form $a + bi$, where a and b are real numbers, what is the value of a ? (Note: $i = \sqrt{-1}$)

(A) $\frac{12}{17}$

(B) $\frac{17}{12}$

(C) $\frac{7}{34}$

(D) $\frac{34}{7}$

Solution

1 – Ace Complex Numbers

Division of exponents on common bases should be subtracted. Rationalize the denominator by multiplying the denominator by its conjugate

$$\left(\frac{4-i}{5-3i}\right)\left(\frac{5+3i}{5+3i}\right)$$

$$\frac{20+12i+\;-5i+\;-3i^2}{25+15i+\;-15i+\;-9i^2}$$

$$\frac{20+7i+\;-3(-1)}{25+\;-9(-1)}$$

$$\frac{24+7i}{34}$$

$$\frac{24}{34}+\frac{7}{34}i$$

$$\frac{12}{17}+\frac{7}{34}i$$

2 – Select Answer

Select answer choice A.

Exponential Decay

3

A radioactive substance decays at a yearly rate of 17 percent. If the initial amount of the substance is 560 grams, which of the following function models the remaining amount of the substance, in grams, *t* years later?

(A) $f(t) = 560(.83)^t$
(B) $f(t) = 560(.17)^t$
(C) $f(t) = .83(560)^t$
(D) $f(t) = .17(560)^t$

Solution

1 – Substitute Abstract With Tangibles

How many grams would be left of the substance if t = 2 years?

After Year 1: 560 (.83) = 465 grams

After Year 2: 465 (.83) = 385 grams

Which answer choice yields 385 grams when t = 2?

A) $f(t) = 560(.83)^2$
B) $f(t) = 560(.17)^2$
C) $f(t) = .83(560)^2$
D) $f(t) = .17(560)^2$

2 – Select Answer **Select answer choice A.**

Equivalent Expressions

4

The expression $\frac{3x+2}{x-4}$ is equivalent to which of the following?

(A) $\frac{3+2}{-4}$

(B) $3-\frac{2}{4}$

(C) $3+\frac{2}{x-4}$

(D) $3+\frac{14}{x-4}$

Solution

1 – Substitute Abstract With Tangibles

Simply plug in x = 2 into the original equation.

$$\frac{3(2)+2}{2-4}$$

$$\frac{8}{-2}$$

$$-4$$

Determine which answer choice yields –4 when x = 2

(A) $\frac{3+2}{-4}$

(B) $3-\frac{2}{4}$

(C) $3+\frac{2}{2-4}$

(D) $3+\frac{14}{2-4}$

2 – Select Answer

Select answer choice D.

Splitting Tasks

5

John and Sally spent 120 hours completing a class project. If Sally spent 30 more hours working on the project than John did, how many hours did John spend?

Solution

1 – Ace Ratios & Percentages

First, imagine that John and Sally completed an equal number of hours on the project.

John – 60 Hours

Sally – 60 Hours

Second, add half of the 30 hours to Sally and half of the 30 hours to John.

John – 45 Hours

Sally – 75 Hours

2 – Fill in Answer

Fill in 45

Use Coefficients Only to Solve

6

The equation $\frac{12x^2+83x-13}{ax+2}=(-4x+3)-\frac{88}{ax+2}$ is true for all values of $x \neq -\frac{2}{a}$, where a is a constant. What is the value of a ?

(A) –6

(B) –3

(C) 3

(D) 6

Solution

1 – Ace Equations

Multiply $-4x+3$ by 1.

$$(-4x+3)\frac{ax+2}{ax+2}$$

$$\frac{-4ax^2-8x+3ax+6}{ax+2}$$

It's not necessary to simplify. Instead, since there are only two terms on both sides of the equations that are to the second power, you can set those coefficients equal to each other.

$$12x^2 = -4ax^2$$
$$12 = -4a$$
$$-3 = a$$

2 – Select Answer

Select answer choice B.

SAP on Tables

7

x	$f(x)$
1	–6
2	–4
3	–2
4	0

The table above shows some values of the linear function f. Which of the following defines f?

(A) $f(x) = x - 8$

(B) $f(x) = 2x - 8$

(C) $f(x) = 2x + 8$

(D) $f(x) = x - 4$

Solution

1 – Substitute Answers in Problem

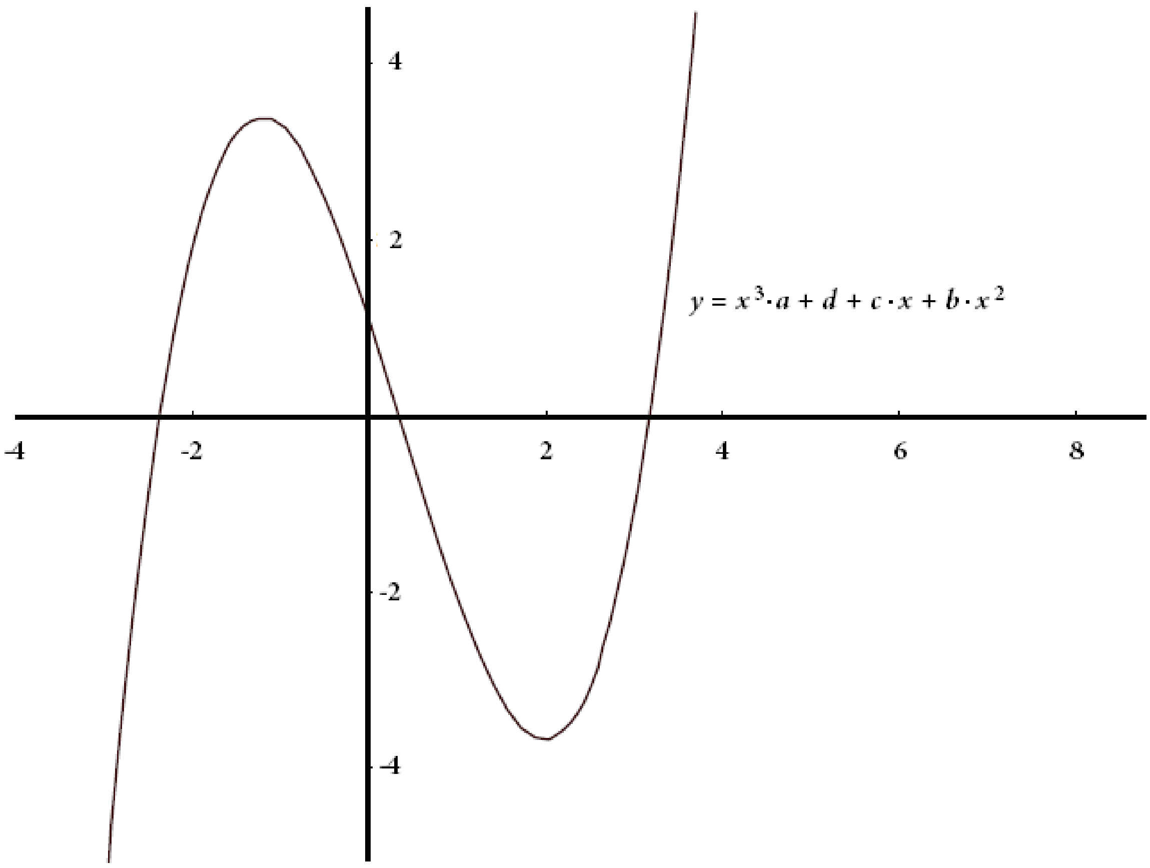
In this case, the answers that you should substitute are in the table. Ask yourself, which of the answer choices yields –6 when I plug in $x = 1$?

2 – Select Answer

Select answer choice B.

Higher Function Graphs

8



The function $f(x)$ is graphed in the xy-plane above. If k is a constant such that $f(x) = k$ has three real solutions, which of the following could be a value of k ?

(A) -4

(B) -2

(C) 4

(D) 6

Solution

1 – Ace Graphs

Remember that $f(x) = k$ is just a horizontal line. Therefore, what horizontal line would intersect the graph three times (three solutions)?

From the appearance of the graph, any horizontal line between the y-values of –3.7 and 3.7 would cross the graph three times.

2 – Select Answer

Select answer choice B.

Adding Proportions/Ratios

9

135 men and women attended a baseball game. The ratio of men to women was 3 to 2.

How many women attended the show?

(A) 5

(B) 17

(C) 34

(D) 51

Solution

1 – Ace Ratios & Percentages

Whenever you are given on a ratio or proportion on the SAT, add the numbers together.

3 + 2 = 5 parts

Divide the actual number by the number of parts.

$$\frac{135}{5} = 17$$ **people per part**

Multiply 17 by the number of parts that women make up of the total ratio.

$$17 \times 2 = 34$$ **women**

2 – Select Answer

Select answer choice C.

Consecutive Numbers

10

The sum of 7 consecutive odd integers is 245. What is the greatest of these 7 integers?

Solution

1 – Ace Ratios & Percentages

Imagine that all of the integers are equivalent.

$$\frac{245}{7} = 35$$

Identify the middle integer.

35, 35, 35, 35, 35, 35, 35

Add/Subtract from the sides of the middle in order to get consecutive odd integers. For example, if you add 2 to the right side, subtract 2 from the left side – this will make sure that the total sum does not change.

29, 31, 33, 35, 37, 39, 41

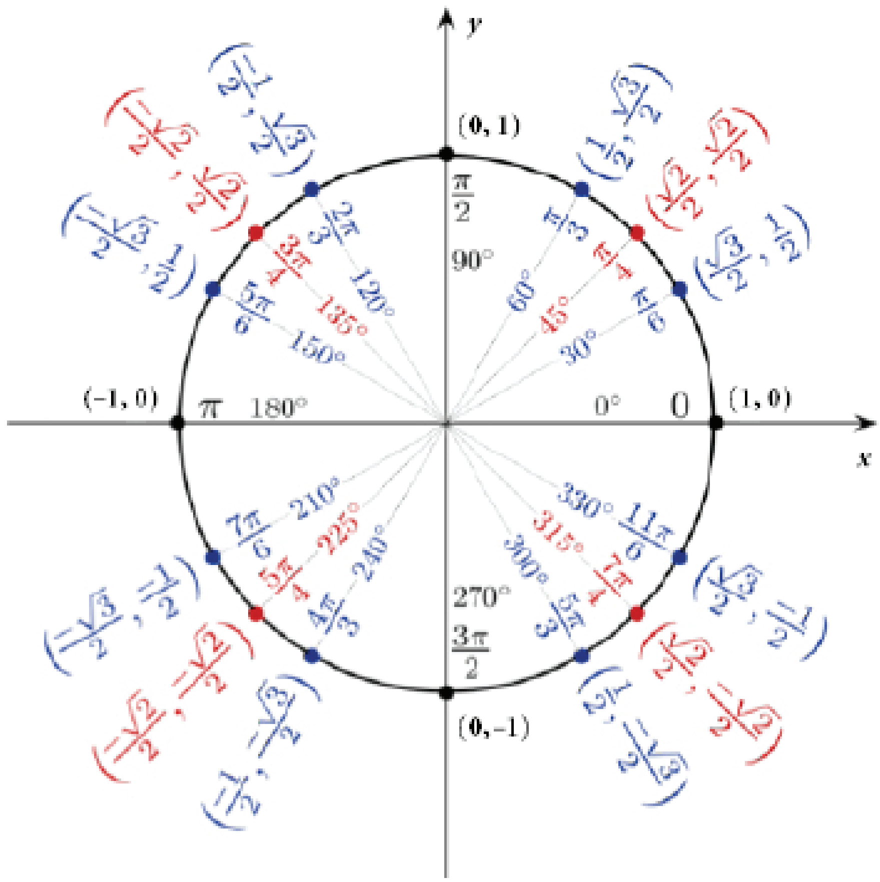
2 – Fill in Answer

Fill in 41

Prep Expert Formula Cheat Sheets

Memorize the formulas and information on this page prior to taking the SAT.

Unit Circle



Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vertex Form of Quadratic Equations

$$y = a(x-h)^2 + k$$

If a is +, the parabola opens upwards

If a is –, the parabola opens downwards

(h, k) represents the vertex of the parabola

Convert Standard Form to Vertex Form

$$y = 2x^2 + 8x + 6$$
$$y = 2(x^2 + 4x + 3)$$
$$y = 2(x^2 + 4x + \_\_) + 6$$
$$y - 6 + 8 = 2(x^2 + 4x + 4)$$
$$y = 2(x + 2)^2 - 2$$

Standard Form of Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

(h, k) represents the center of a circle

Convert Equation to Standard Circle Equation

$$3x^2 + 3y^2 - 12x + 24y - 81 = 0$$
$$3(x^2 + y^2 - 4x + 8y) = 81$$
$$3(x^2 - 4x + \_\_) + 3(y^2 + 8y + \_\_) = 81$$
$$4(x^2 - 4x + 4) + 3(y^2 + 8y + 16) = 81 + 12 + 48$$
$$3(x - 2)^2 + 3(y + 4)^2 = 141$$

Simple Interest

$$A = P(1 + rt)$$

A = amount at the end of t years

P = original amount of money deposited (or "principal")

r = interest rate

t = time in years

Compound Interest

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

A = amount at the end of t years

P = original amount of money deposited (or "principal")

r = interest rate

t = time in years

m = # of compounding periods per year